IEEE Magnetics Summer School 2019

Simulation and theory on magnetism

Part 2

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Outline (Part 2)

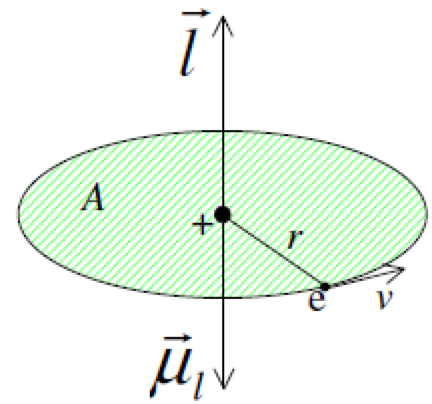
1. Spin-orbit coupling origin
    1. SOC origin
    2. Spin Hall effect, Inverse spin Hall effect, Rashba effect
2. Rashbaness of FM/HM bilayers
    1. Orbital moment
    2. Signatures of Rashbaness in FM/HM bilayers
3. Interfacial magnetic properties
    1. Interfacial or surface magnetocrystalline anisotropy energy
    2. Interfacial Dzyaloshinskii-Moriya interaction & Chiral Damping
    3. Angular dependence of spin-orbit torque (SOT)

1. Spin-orbit coupling origin

Spin-orbit coupling

- The electron
  - Spin (angular momentum) s (~ self-rotation of electron)
  - Orbital angular momentum l (~ rotation of electron around nuclei)
- For each angular momentum, there is an associated magnetic moment, μ_l and μ_s
- Spin-orbit coupling (SOC) = Interaction between these two magnetic moments

Orbital magnetic moment μ_l



- Electron orbit ➔ current ➔ magnetic moment
- μ_l = (current) X (area of the loop) = I A
- Orbit period = T = $2\pi/\omega$
- I = (charge)/(Time) = -e/T = $-e\omega/(2\pi)$
- μ_l = I A = $-e\omega/(2\pi)$ X (πr^2) = $-e\,\omega\, r^2/2$
- Angular momentum $|\boldsymbol{l}| = |\mathbf{r} \times \mathbf{p}| = m_0 v r = m_0 \omega r^2$
- ➔ $\boldsymbol{\mu}_l = -e/(2 m_0)\, \boldsymbol{l}$
- Note1: $\boldsymbol{\mu}_l$ and $\boldsymbol{l}$ are in opposite direction ($e < 0$)
- Note2: If $|\boldsymbol{l}| = h/(2\pi)$ = hbar [ground state of Bohr model]

➔ μ_B = e hbar / $(2 m_0)$: Bohr magneton = unit of magnetic moment

Spin magnetic moment μ_s

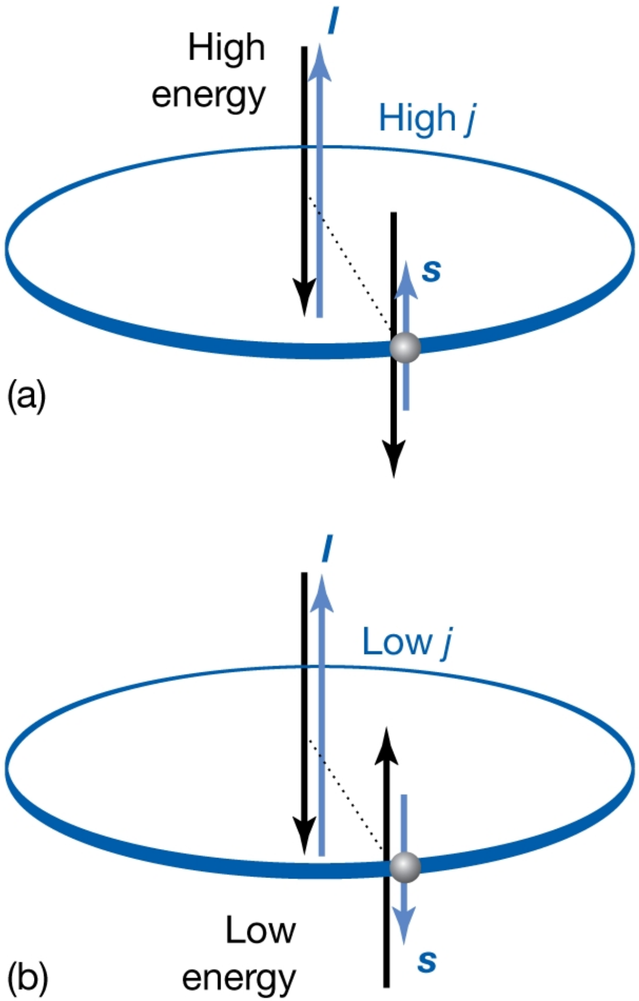
- (electron) Spin = intrinsic angular momentum with quantum number s = ½
- Justified by Stern-Gerlach experiment + Dirac's relativistic theory
- ➔ $\boldsymbol{\mu}_s = -g\, e/(2\, m_0)\, \boldsymbol{s}$
- g = Landé g-factor = 2.002319 for an atom (usually larger than 2 in solid)
- Note1: g = 1 classically (Bohr model), g = 2 comes from Dirac's relativistic quantum theory
- Small corrections from quantum electrodynamics

Einstein-de-Haas effect vs Barnett effect

- Einstein-de-Hass ➔ current pulse in solenoide ➔ B field ➔ sample M increases ➔ sample rotates ➔ M ~ angular momentum
- Barnett effect ➔ inverse effect of Einstein-de-Hass

Magnetic energy ➔ SOC

- From "nucleus" point-of-view = electron rotates around me and generates a magnetic field
- From "electron" point-of-view = nucleus rotates around me and gerenrates a magnetic field = relativistic electron's orbital motion ➔ $\mathbf{B}_l$
- E = $-\boldsymbol{\mu}_s \cdot \mathbf{B}_l$ ➔ SOC
- Note: Also an energy for "nucleus" (because nucleus has its own spin). But it is small and contributes to hyperfine structure



SOC in Dirac equation

- 1/c expansion of Dirac eq.

$$\hat{H} = \underbrace{\frac{\hat{p}^2}{2m} + eV}_{\text{non-relativistic}} + \underbrace{\frac{\hat{p}^4}{8m^2c^2}}_{\text{K.E. correction}} + \underbrace{\frac{\hbar^2}{8m^2c^2}\nabla^2 V}_{\text{Darwin term}} + \underbrace{\frac{\hbar}{4m^2c^2}\vec{\sigma}\cdot(\vec{\nabla}V \times \hat{\mathbf{p}})}_{\text{SOI}}$$

- 4^{th} term = SOC : σ = spin, V = potential

$$U = \frac{\hbar}{4m^2c^2}\boldsymbol{\sigma}\cdot(\nabla V \times \mathbf{p})$$

When an electron passes (**p**) through the region having potential gradient (= E field), its spin (σ) experiences an effective magnetic field (~ **E** x **p**).

$$\nabla V = \frac{\mathbf{r}}{r}\frac{dV}{dr}$$

$$U = \frac{\hbar}{4m^2c^2}\frac{1}{r}\frac{dV}{dr}\boldsymbol{\sigma}\cdot(\mathbf{r}\times\mathbf{p}) = \frac{\hbar}{4m^2c^2}\frac{1}{r}\frac{dV}{dr}\boldsymbol{\sigma}\cdot\mathbf{L}$$

Why SOC?

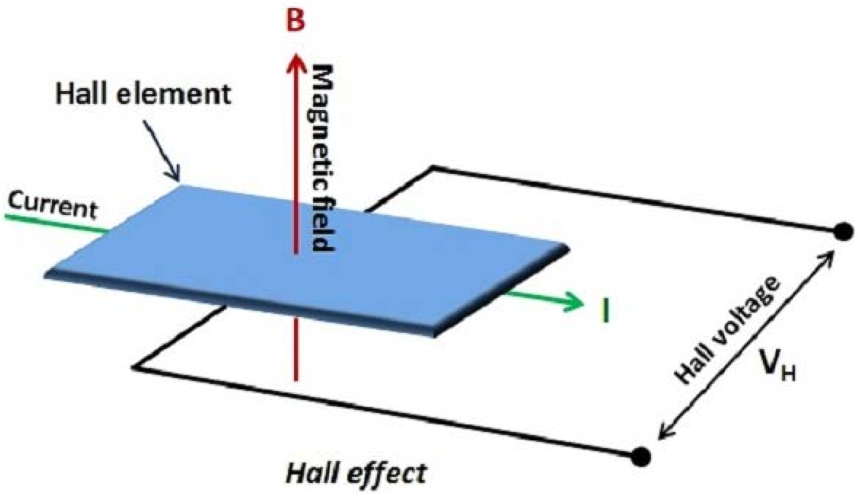
- Electron spin passing through E field feels an effective magnetic field == SOC
- Electron spin rotates around the magnetic field and then is aligned

➔ Spin(-polarized) current

- Spintronics ➔ Do something with spin current
- How to generate spin current = effective magnetic field
  - True magnetic field (but require huge one)
  - Passing through a magnet (exchange field >> 10 T)
  - Use SOC (even without FM) ➔ spin Hall effect
- What we can do with spin current
  - Spin transfer torque ➔ manipulation of local magnetization
  - Spin current <-> Charge current conversion by SOC ← Onsager reciprocity ➔ Spin Hall effect and Inverse spin Hall effect

Spin Hall effect (SHE)

- Charge current (Je) is converted to spin current (Js) by SOC ~ spin version of the Hall effect



$$\mathbf{V}_H = R_H \mathbf{B} \times \mathbf{J}_e \Leftrightarrow \mathbf{J}_S = \theta_{SH} \boldsymbol{\sigma} \times \mathbf{J}_e$$

A simple picture of SHE

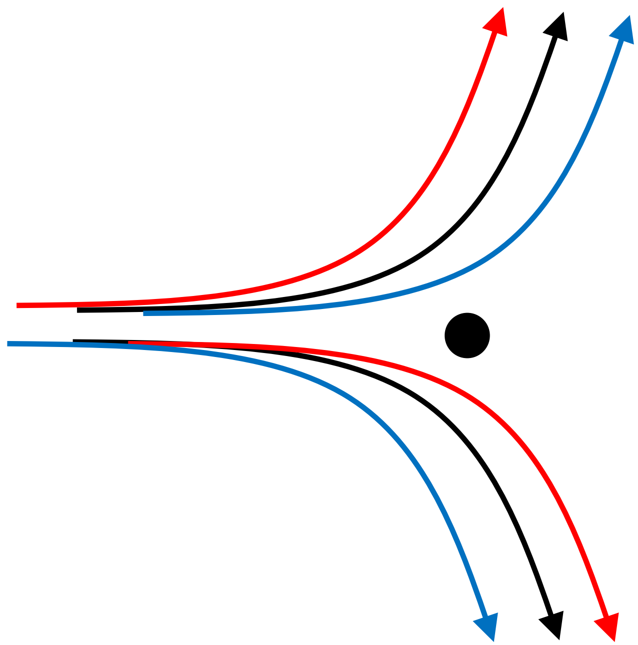
- Repulsive scatterer V(r) [dV/dr < 0]
  - Total scattering potential

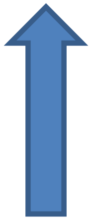
$$V_{\text{eff}} = V(r) + \frac{1}{2m^2c^2r}\frac{dV}{dr}\mathbf{L}\cdot\mathbf{S}$$

Spin Up

Spin Down

⊙





Charge current ➔

perpendicular Spin current due to L-S coupling

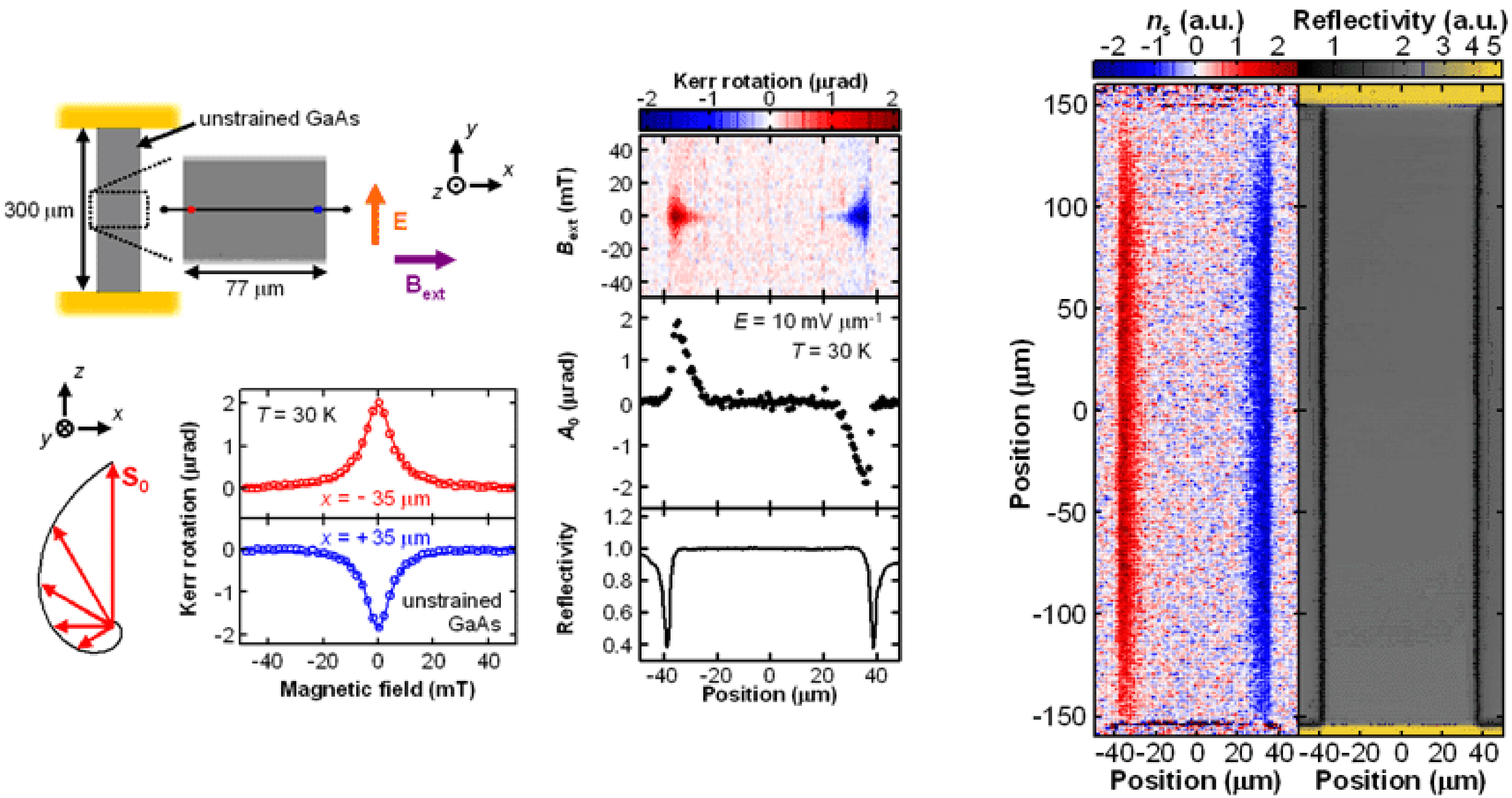
Spin Hall angle $\theta_{SH} = J_S/J_C$

⊗

Courtesy of Prof. H.-W. Lee

Experiment: SHE

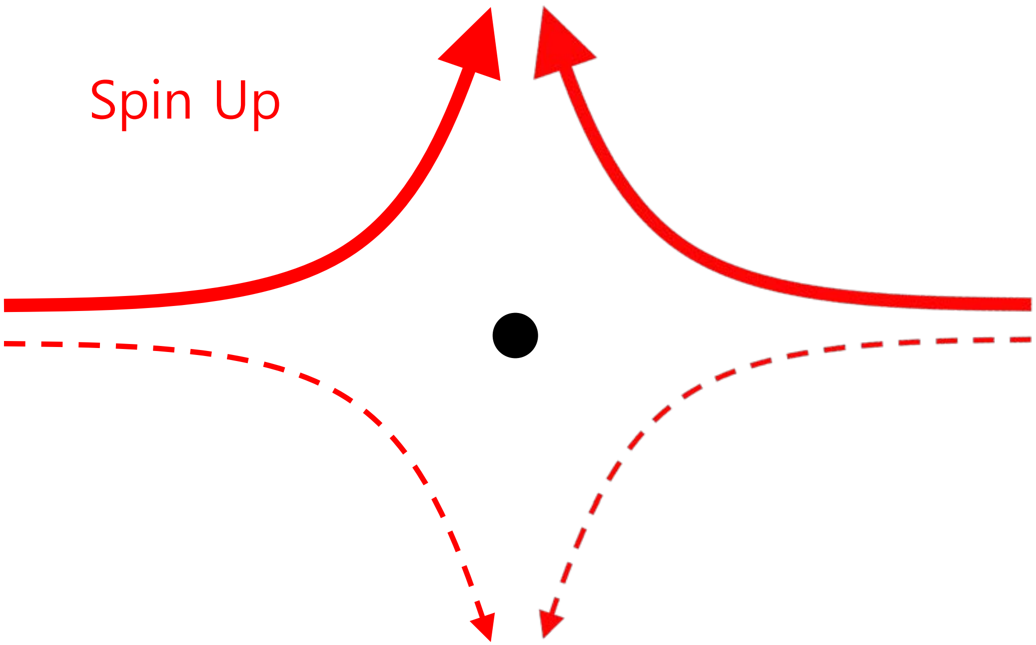
- Y. K. Kato *et al.*, Science **306**, 1910 (2004)

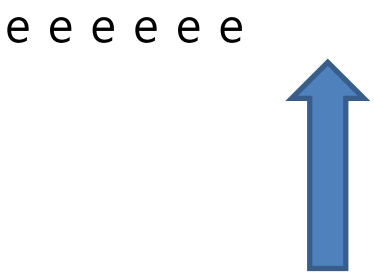


Inverse Spin Hall effect (ISHE)

- Spin current (Js) is converted to charge current (Je)

$$\mathbf{J}_e = \theta_{SH} \boldsymbol{\sigma} \times \mathbf{J}_s$$



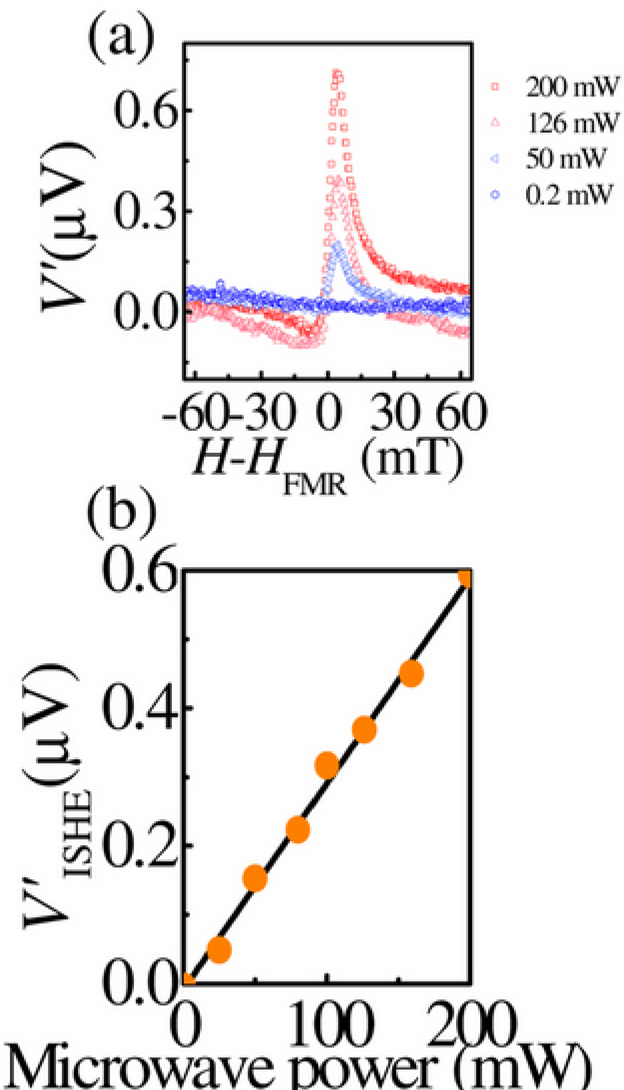


Spin current ➔ perpendicular Charge current due to L-S coupling

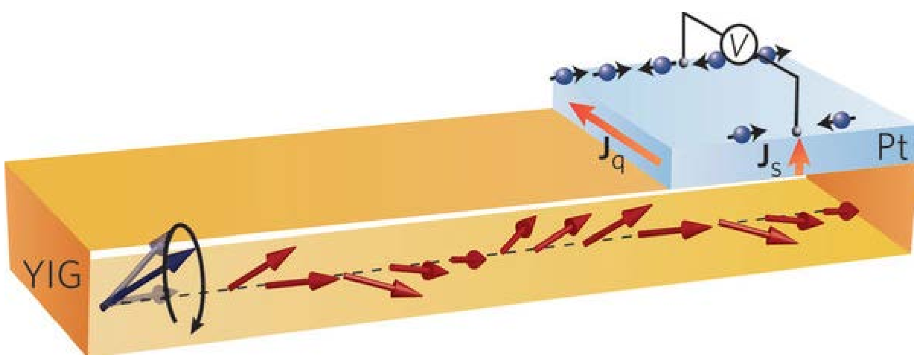
Spin Hall angle $\theta_{SH} = J_S/J_C$

Experiment: ISHE

- E. Saitoh *et al.*, APL **88**, 182509 (2006)



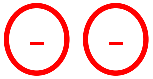
Sensing the Spin Seebeck effect



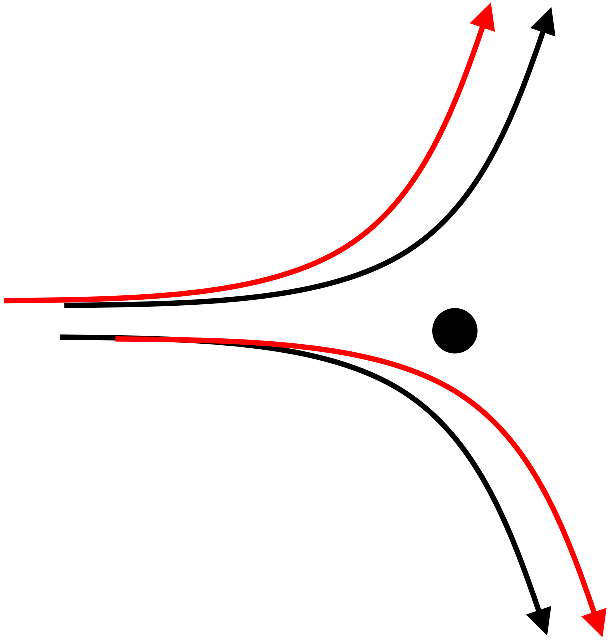
Anomalous Hall effect (AHE)

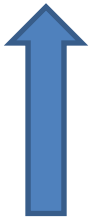
- Repulsive scatterer V(r) [dV/dr < 0]
  - Total scattering potential

$$V_{\text{eff}} = V(r) + \frac{1}{2m^2c^2r}\frac{dV}{dr}\mathbf{L}\cdot\mathbf{S}$$



Spin Up ONLY





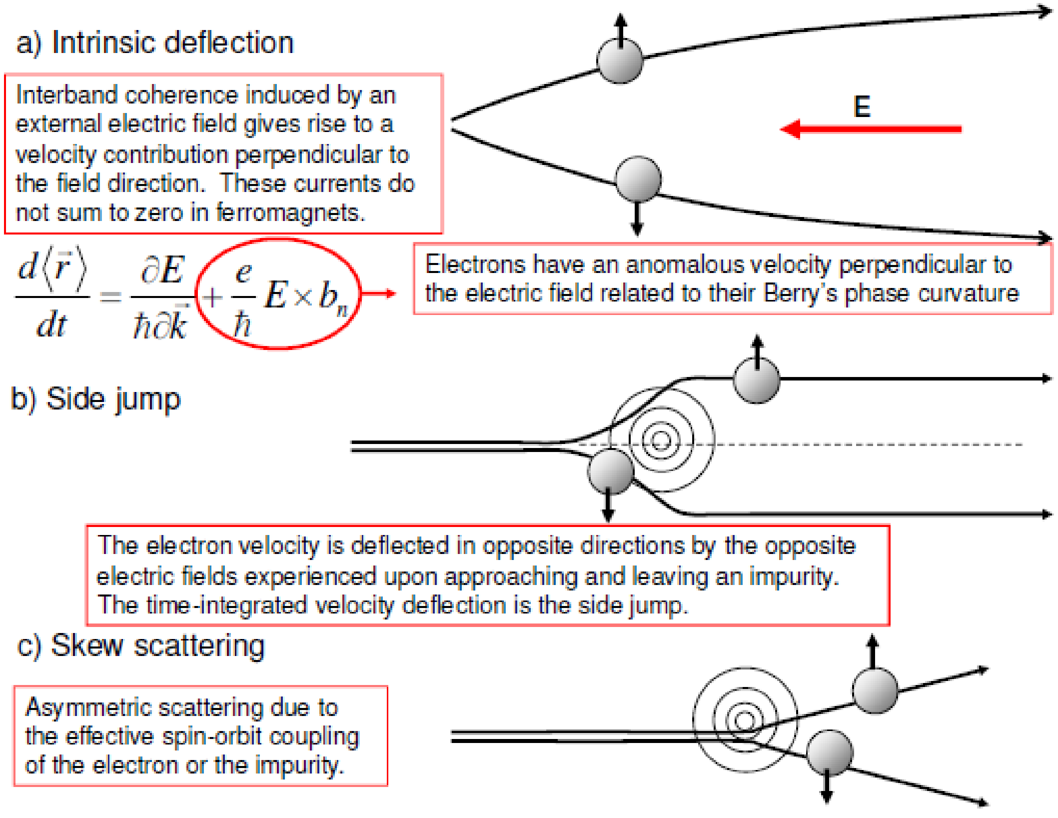
Longitudinal charge current ➔

Transverse Hall voltage

~ SHE

Three mechanisms of AHE

- Nagaosa et al. RMP '09



$$\sigma_{xy} \propto \sigma_{xx}^{0}$$

$$\sigma_{xy} \propto \sigma_{xx}^{1}$$

Note 1: "Intrinsic" and "side-jump" are not easy to be separated in experiment

Note 2: "Skew" is dominant for very good metal.

2. “Rashbaness” of FM/HM bilayers

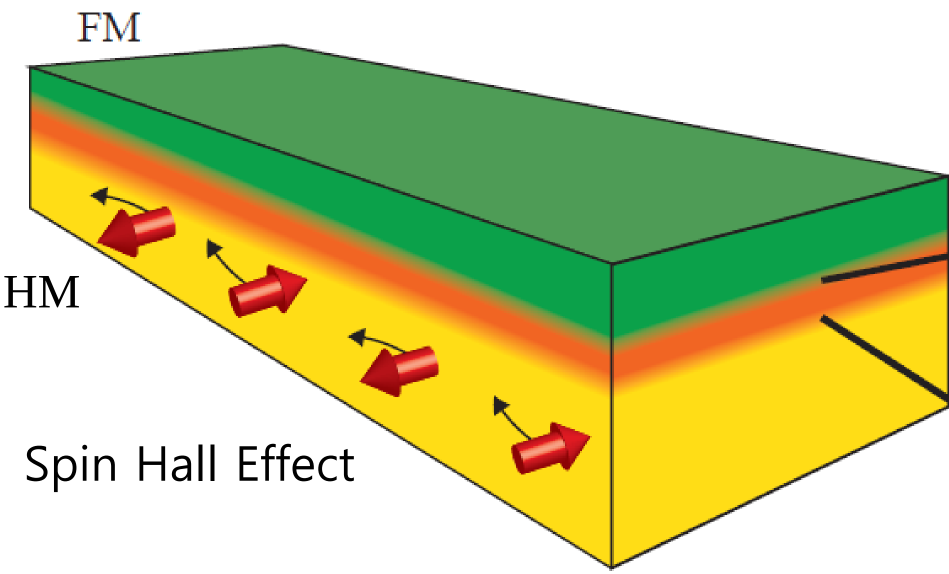
2-1. Orbital moment for RSOC

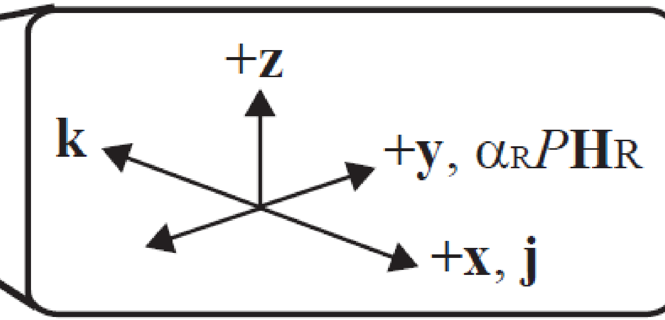
2-2. Signatures of Rashbaness in FM/HM bilayers

Ferromagnet (FM)/Heavy metal (HM) bilayer

Interfacial spin-orbit coupling

$$H_{\mathrm{RSOC}} = \frac{\alpha_{\mathrm{R}}}{\hbar}\boldsymbol{\sigma}\cdot\mathbf{p}\times\hat{\mathbf{z}}$$





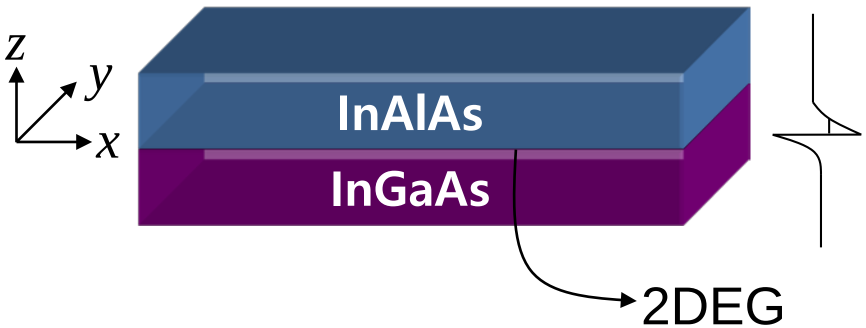
- **Effects of Interfacial (Rashba) SOC**
  - **Field-like spin torque**: Manchon & Zhang, PRB 08; Obata & Tatara, PRB 08; Matos-Abiague & Rodriguez-Suarez, PRB 09
  - **Damping-like spin torque**: Wang & Manchon, PRL 12; Kim, Seo, Ryu, KJL, HWL, PRB 12; Pesin & MacDonald, PRB 12; Kurebayashi et al., Nat. Nano. '14
- Experiment: Miron et al., Nat. Mat. 10, 11; Nature 2012 (Pt|Co|AlOx)

A simple picture of Rashba effect

- Relativistic correction

$$H_{so} = -\boldsymbol{\mu}_B \cdot \mathbf{B}_{eff} = -\frac{1}{2}\left(\frac{e\hbar}{2m}\boldsymbol{\sigma}\right)\cdot\left(-\frac{1}{c^2}\mathbf{v}\times\mathbf{E}\right) = \frac{\hbar}{4m^2c^2}\sigma\cdot\mathbf{p}\times\nabla V(\mathbf{r})$$

- Structural inversion asymmetry in heterostructures



$$H_R = \frac{\alpha_R}{\hbar}\boldsymbol{\sigma}\cdot(\mathbf{p}\times\hat{\mathbf{z}})$$

$\Rightarrow$

$$\textbf{Let } \mathbf{p} = \hbar k_F \hat{\mathbf{x}}$$

$$H_R = \alpha_R k_F \boldsymbol{\sigma}\cdot(\hat{\mathbf{x}}\times\hat{\mathbf{z}}) = -\boldsymbol{\sigma}\cdot\alpha_R k_F \hat{\mathbf{y}}$$

$$\mathbf{E}_R = -\boldsymbol{\sigma}\cdot\mathbf{B}_{eff} \Rightarrow \mathbf{B}_{eff} \propto \alpha_R k_F \hat{\mathbf{y}}$$

Rashba SOC generates spin density along **y**.

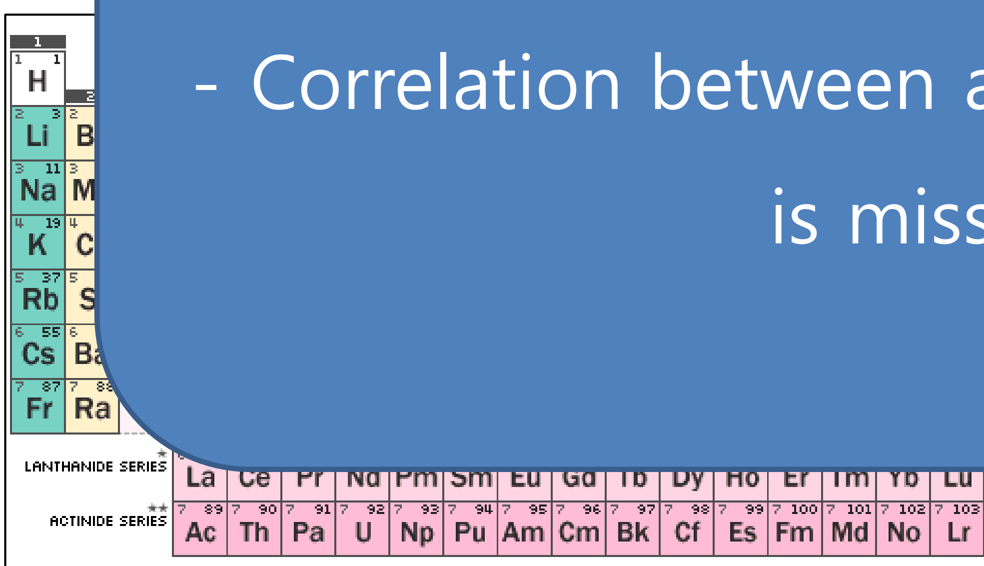
Courtesy of H.-W. Lee

RSOC strength I

- Ast *et al*., Phys. Rev. Lett. 98, 186807 (2007)

TABLE I. Selected materials and parameters characterizing the spin splitting: Rashba energy of split states E_R, wave number offset k_0...

InGaA...
Ag(11...
Au(11...
Bi(111...
Bi/Ag...



- "Relativistic magnetic" field + Zeeman energy

$\ldots V(\mathbf{r})$

$\ldots) \cdot \hat{\mathbf{z}}$

...rence

"Mysteries" of RSOC strength

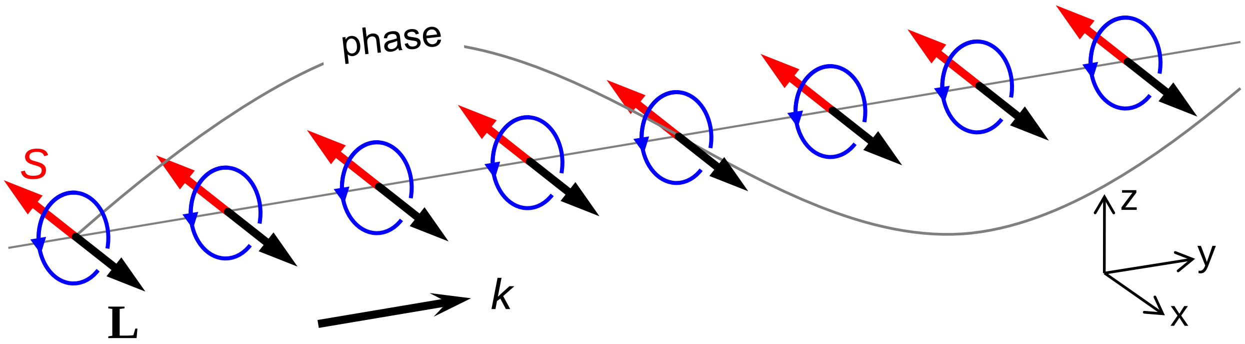
- "Relativistic magnetic field + Zeeman energy" interpretation ➔ Too small α_R
- Correlation between atomic SOC and α_R is missing

RSOC strength II

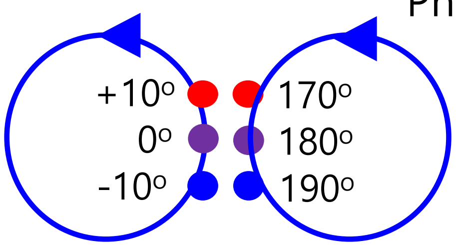
Courtesy of H.-W. Lee

S. R. Park et al., PRL 107, 156803 (2011)

- Orbital angular momentum



If NOT Bloch state



Phase difference

160° ➔ slightly constructive (electron density = $\sin^2(-20^o)$)

180° ➔ destructive (electron density = 0)

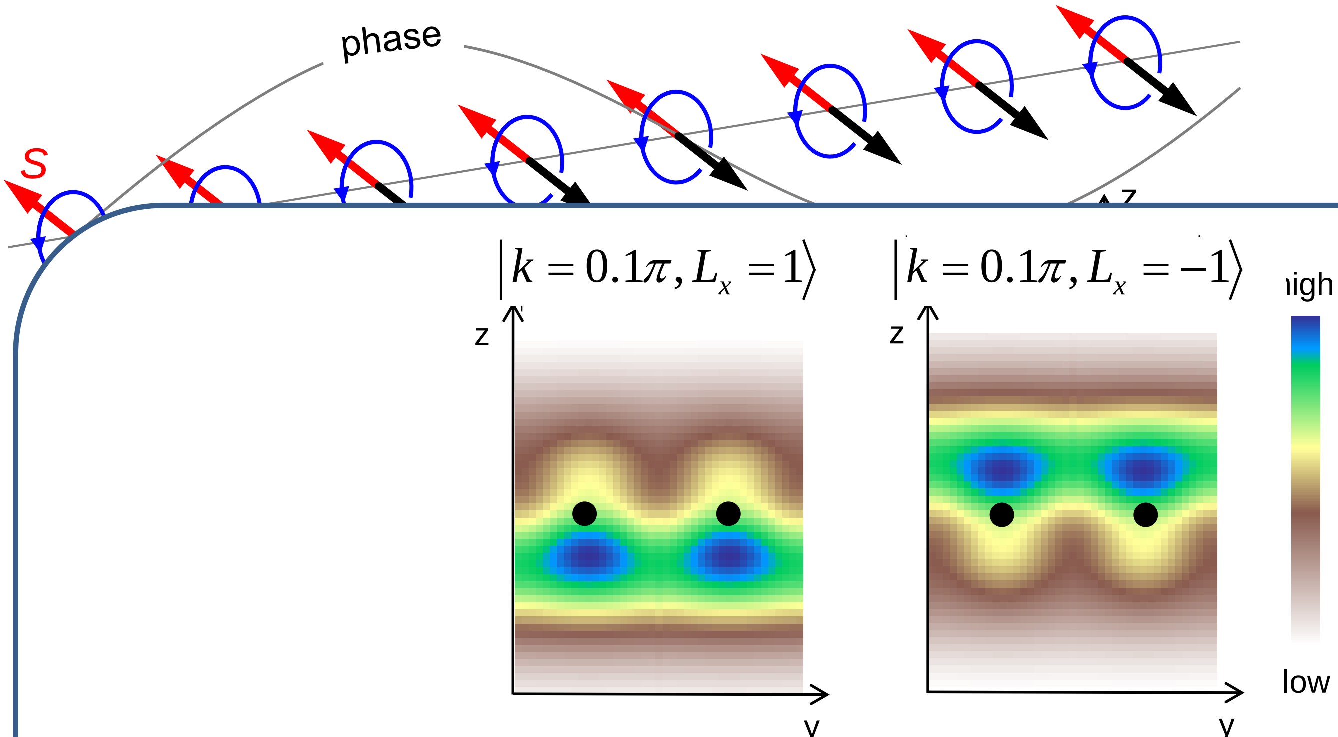
200° ➔ slightly constructive (electron density = $\sin^2(+20^o)$)

RSOC strength II

Courtesy of H.-W. Lee

S. R. Park et al., PRL 107, 156803 (2011)

- Orbital angular momentum



$\Delta E \propto (\mathbf{L} \times \mathbf{k})\cdot$(inversion symmetry breaking vector)

when SOC is strong, **S** is aligned along **L** ➔

$\propto \lambda_{SOC}\,(\mathbf{S} \times \mathbf{k})\cdot$(inversion symmetry breaking vector)

α_R : Magnitude estimation

"Relativistic magnetic" field
+ Zeeman energy

$$H_{so} = \frac{\hbar}{4m^2c^2}\sigma \times \mathbf{p} \cdot \nabla V(\mathbf{r})$$

$$H_R = \frac{\alpha_R}{\hbar}(\sigma \times \mathbf{p}) \cdot \hat{\mathbf{z}}$$

$$\alpha_R = \frac{1}{4}\left(\frac{\hbar}{mc}\right)^2 \langle \nabla V \rangle$$

$$\sim \frac{1}{4}(\alpha a_0)^2 \frac{\text{Work function difference}}{\text{atomic spacing}}$$

$$\sim \frac{1}{4}\alpha^2 \times \text{eV} \cdot \text{A} \sim 10^{-4}\ \text{eV} \cdot \text{A}$$

a_0: Bohr radius

α : Fine structure constant 1/137

Orbital-induced electric dipole
+ Atomic spin-orbit coupling

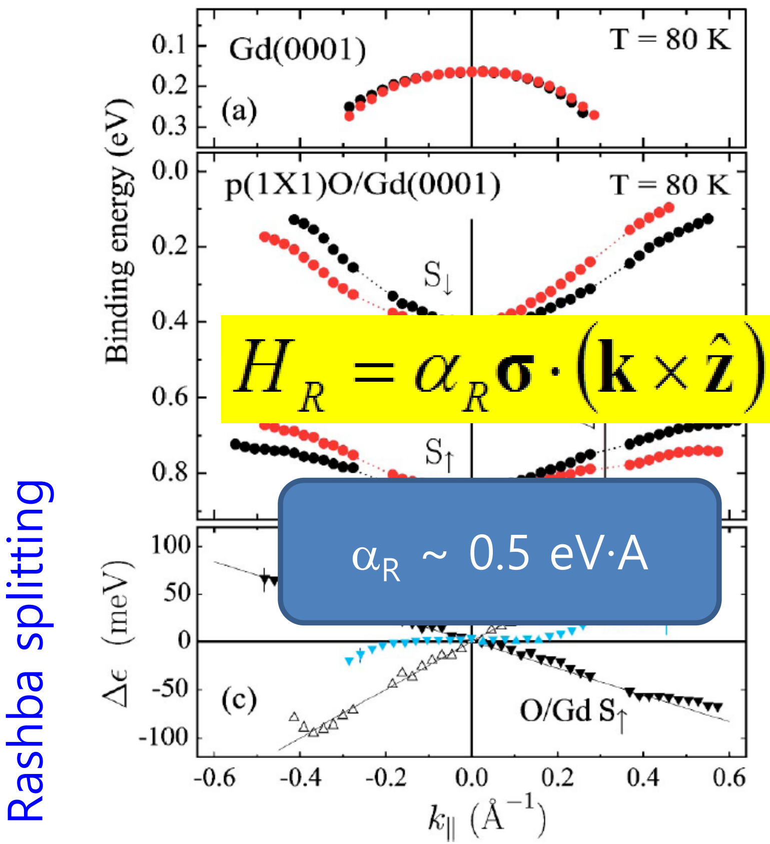
- $\Delta E \propto$ (Electric dipole energy)
  - Relevant length scale
    - a_0, ~ A
  - λ_{SOC} ~ 0.1 – 1 eV
- α_R ~ 0.1 – 1 eV·A
  - Similar to experimental values
  - Explains correlation with atomic number Z

Courtesy of H.-W. Lee

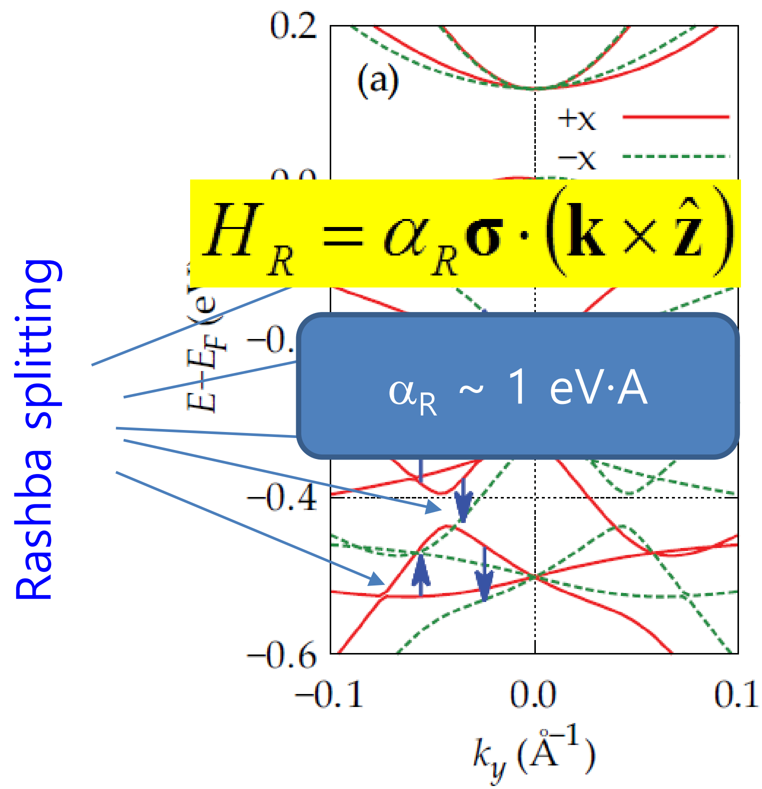
Interfacial spin-orbit coupling (ISOC)

Heavy Metal (HM) / Ferromagnet (FM) / Oxide trilayers

- **Oxide/FM interface**
- Gd/O; angle-resolved photoemission experiment (Krupin PRB 2005)

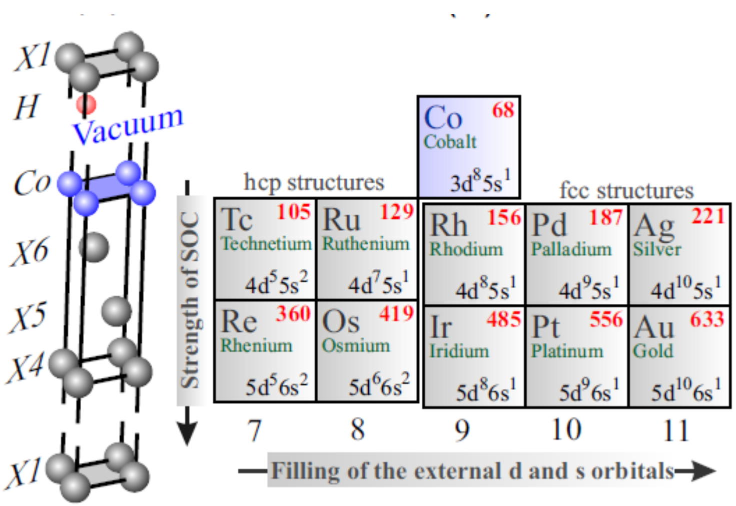


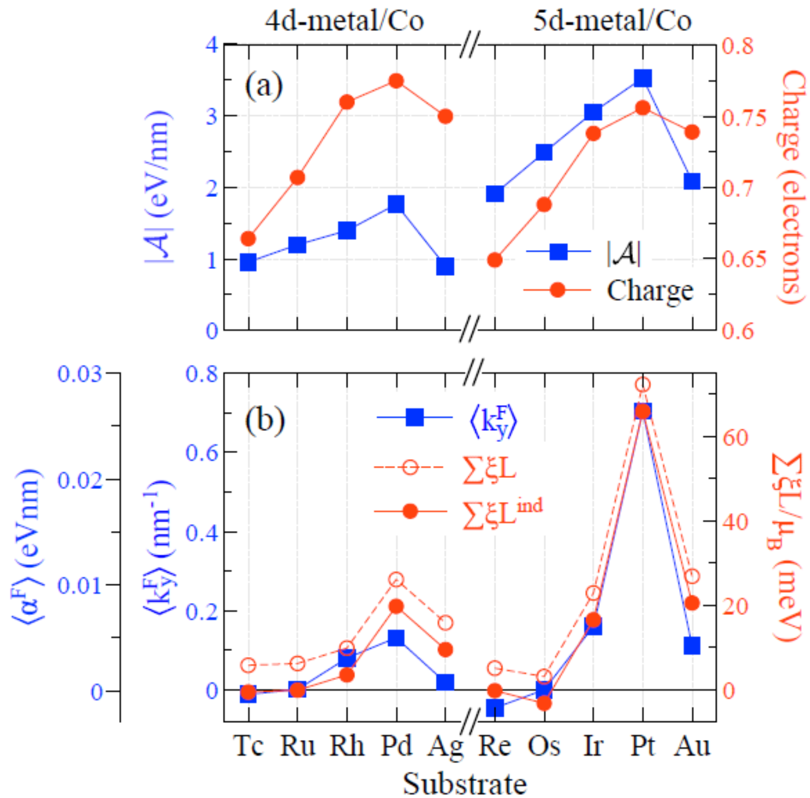
- **FM/HM interface**
- Co/Pt; ab initio calculation (Park PRB 2013)

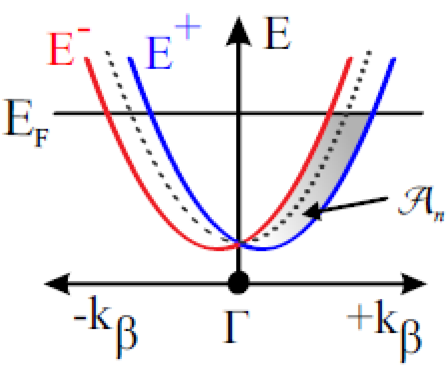


Rashbaness of various Co/HM bilayers (1)

Grytsyuk, ..., KJL, and Manchon, PRB 93, 174421 (2016)







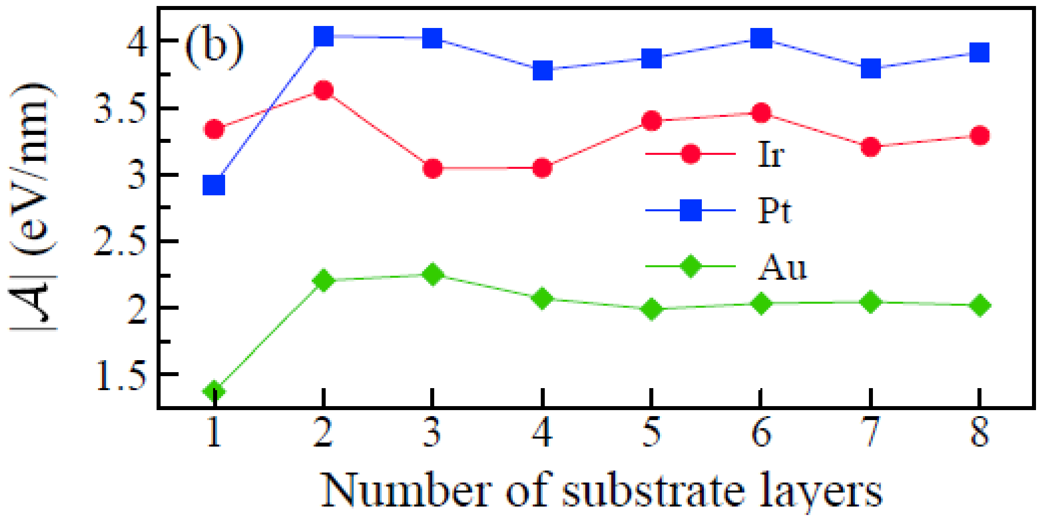
Top right = charge transfer from HM-d to Co-d
Bottom right = induced orbital moment

All Co/HM bilayers show "Rashbaness" A.

Orbital magnetization is a measure of "Rashbaness"

Rashbaness of various Co/HM bilayers (2)

- In FM/HM bilayers, many experimental reports find the effective spin Hall angle scales with $\exp(-t_{HM}/\lambda_{HM})$ where λ_{HM} is commonly identified as spin diffusion length of HM.
- The reported spin diffusion length ~ 1 nm regardless of HM (Pt, Ta, W, etc)
- The resistivity of Pt << resistivity of Ta or W ➔ very different m.f.p.
- Spin diffusion length ~ $(\text{m.f.p.})^{1/2}$

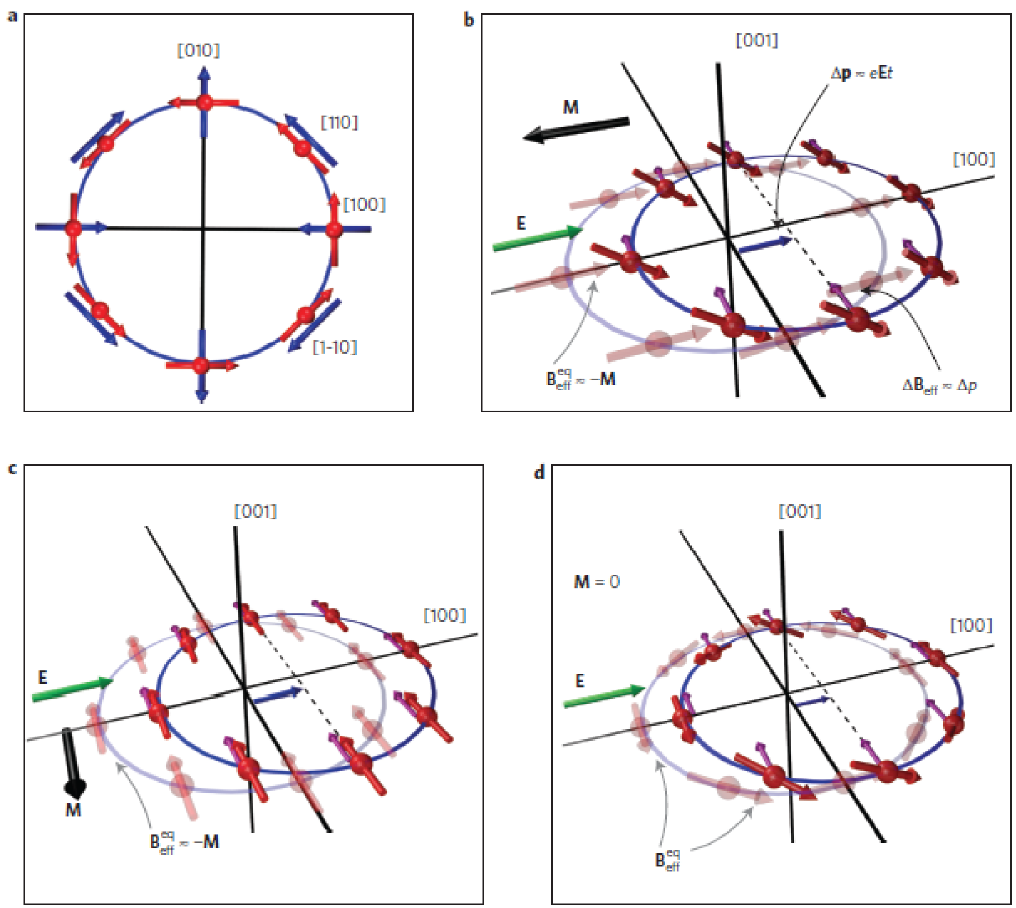


The "Rashbaness" A saturates in 2~3 monolayers of HM ~ 1 nm

Damping-like SOT from Berry curvature

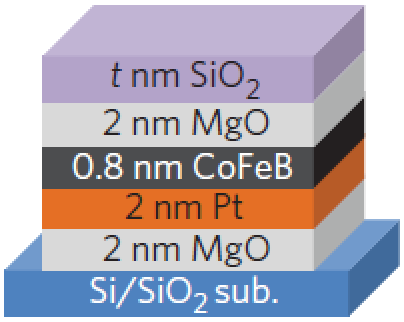
Kurebyayshi et al. Nat Nano ‘13

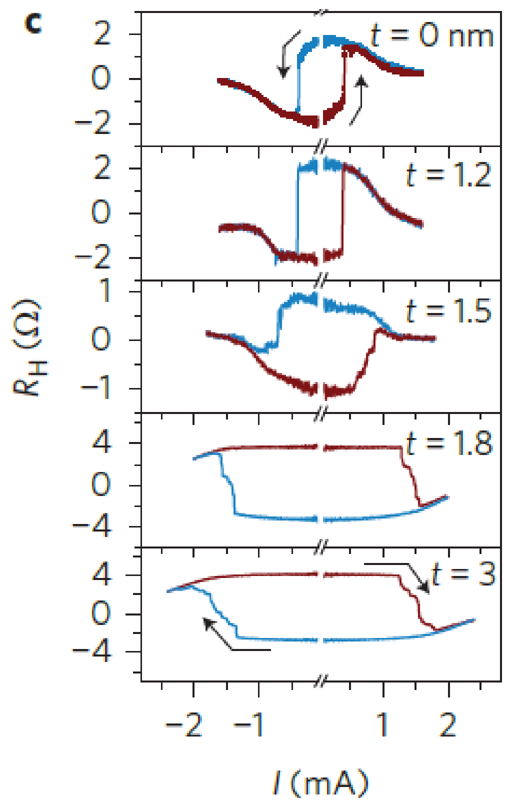
- (Ga, Mn)As → no HM ➔ no bulk spin Hall effect
- Well-known band structure ➔ Rashba SOC
- Damping-like SOT ~ Field-like SOT ➔ Both SOT components well described by Rashba model

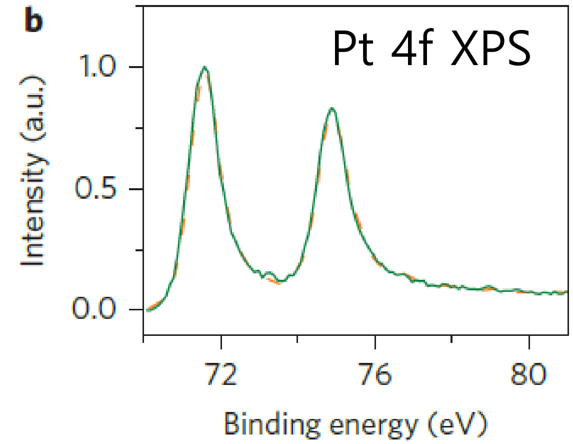


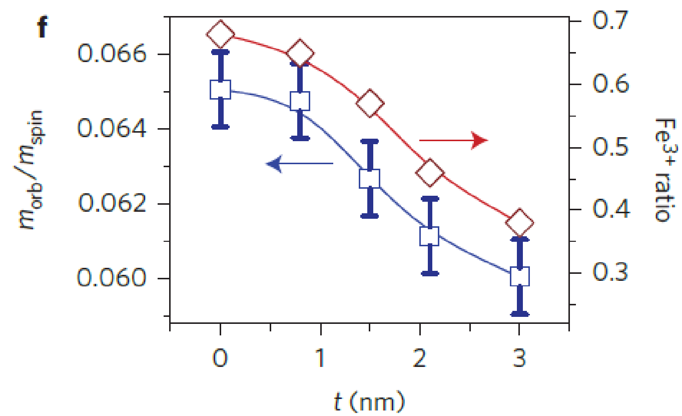
SOT engineering via oxygen manipulation

Qiu, …, KJL, Yang, Nat Nano '15









- SiO thickness controls the oxidation of CoFeB layer ➔ Change in the sign of damping-like SOT (note: Pt is not oxidized)
- Close correlation with the orbital moment

3. Interfacial magnetic properties

3-1. Magnetocrystalline anisotropy energy

3-2. Chiral derivative ➔ Interfacial DMI and Chiral damping

3-3. Angular dependence of SOT

MagnetoCrystalline Anisotropy Energy (MCA)

- Bruno model (PRB 39, 865 (1989)) ➔ MCA ~ orbital magnetic moment anisotropy

$$MCA \sim \mu_L^{\perp} - \mu_L^{//}$$

MCA

$$H = \frac{\hbar^2 k^2}{2m} + J_{sd}\boldsymbol{\sigma}\cdot\mathbf{m} + \alpha_R \boldsymbol{\sigma}\cdot(\mathbf{k}\times\mathbf{z}) \;\&\; \mathbf{m} = (m_x, 0, m_z)$$

$$E_k^{\pm} = \frac{\hbar^2 k^2}{2m} \mp \left| J_{sd}\mathbf{m} + \alpha_R(\mathbf{k}\times\mathbf{z}) \right| \approx \frac{\hbar^2 k^2}{2m} \mp \left(J_{sd} + \alpha_R k_y m_x + \frac{\alpha_R^2 (k^2 - k_y^2 m_x^2)}{2J_{sd}} \right)$$

$$E(m_x) = \int \frac{dk^2}{(2\pi)^2}(E_k^+ + E_k^-) \Rightarrow MCA = E(m_x = 1) - E(m_x = 0) \approx C\alpha_R^2$$

Shi et al. PRL '07

OM

$$\boldsymbol{\mu} = i\frac{e}{2\hbar}\sum_{n\mathbf{k}} \left\langle \nabla_{\mathbf{k}} u_{n\mathbf{k}} \right| \left[\varepsilon_n(\mathbf{k}) - \hat{H}_0(\mathbf{k}) \right] \nabla_{\mathbf{k}} u_{n\mathbf{k}} \rangle \sim \alpha_R^2 M_z$$

Rashba model is able to capture the core physics of MCA.

Interfacial magnetocrystalline anisotropy

Kim, KJL, … PRB 94, 184402 (2016)

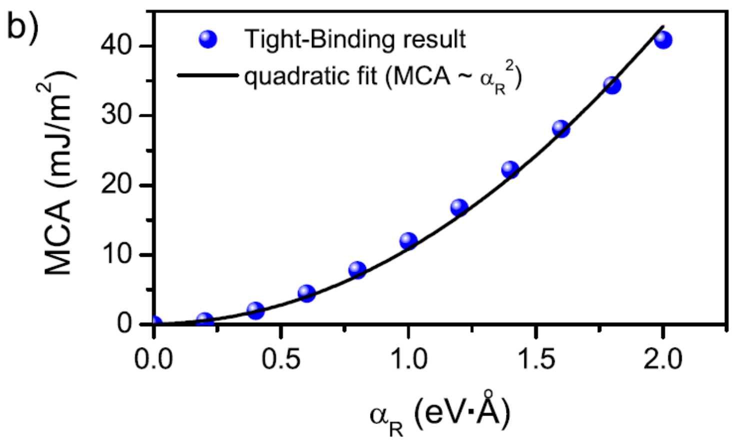
Free electron model (parabolic band)

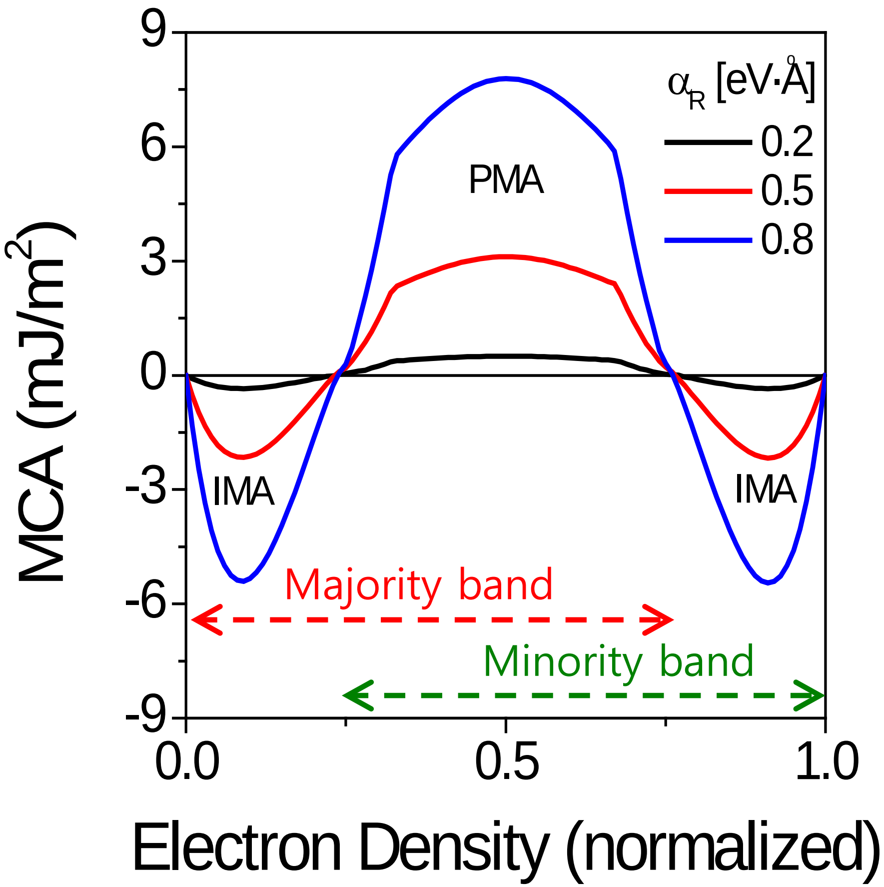
➔ MCA vanishes once minority band is occupied.

Tight-binding model

J_{sd} = 1 eV, $m_e = m_0$

Square lattice





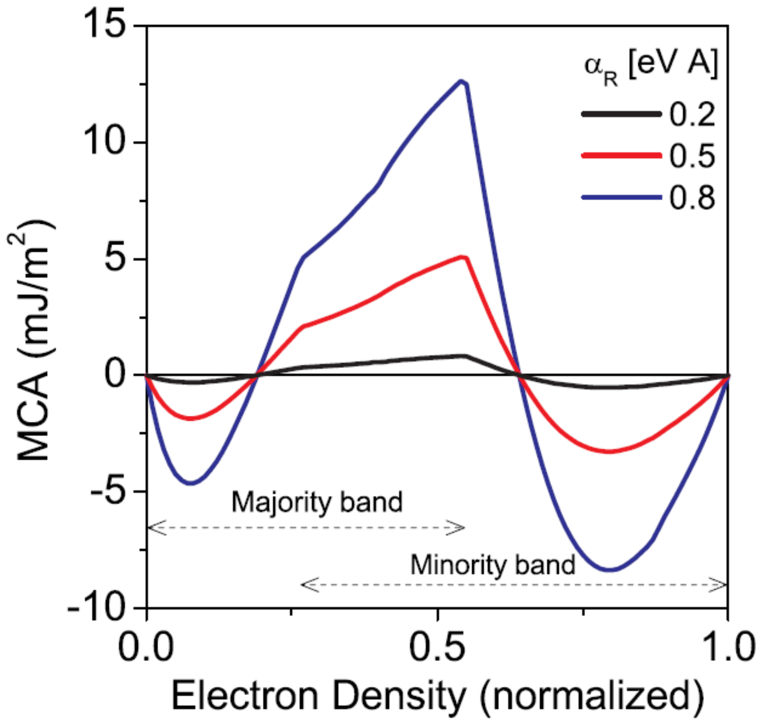
- RSOC ➔ Interfacial MCA
- Band filling determines PMA / IMA

Interfacial magnetocrystalline anisotropy

Tight-binding model

J_{sd} = 1 eV, $m_e = m_0$

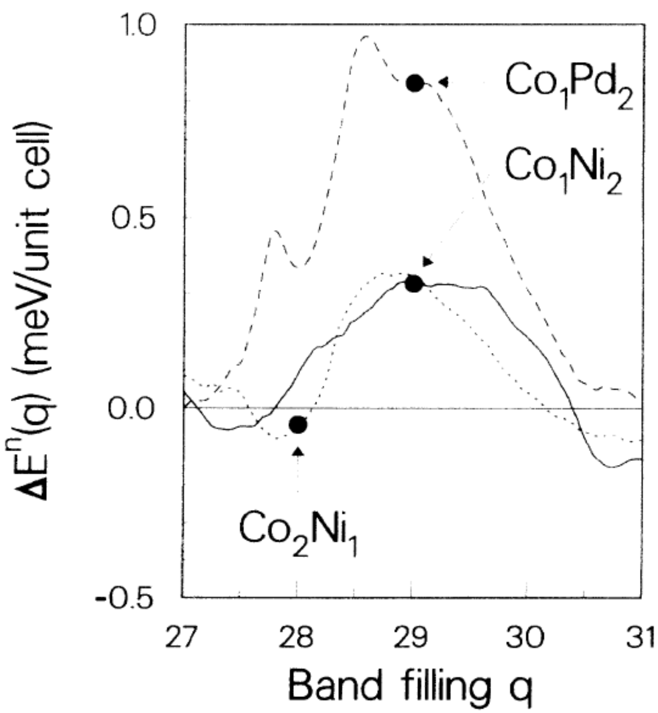
Triangular lattice



First principles

Daalderop et al. PRL 68, 682 (1992).

Co/Ni, Co/Pd multilayers (fcc 111)



- Simple Rashba model captures the core effect of MCA

Interfacial spin-orbit spin torques

"Magnetic interfaces (inversion asymmetry) + Spin-Orbit coupling"

Field-like torque

$$\mathbf{T}_\mathrm{f} = \frac{2\alpha_\mathrm{R} m_e}{\hbar^2} b_j \hat{\mathbf{y}} \times \hat{\mathbf{m}}$$

Damping-like torque (Slonczewski-like)

$$\mathbf{T}_\mathrm{d} = -\frac{2\alpha_\mathrm{R} m_e}{\hbar^2} \beta b_j \hat{\mathbf{m}} \times \left(\hat{\mathbf{y}} \times \hat{\mathbf{m}} \right)$$

Adiabatic torque

$$\mathbf{T}_\mathrm{adia} = b_j \partial_x \hat{\mathbf{m}}$$

Nonadiabatic torque

$$\mathbf{T}_\mathrm{non} = -\beta b_j \hat{\mathbf{m}} \times \partial_x \hat{\mathbf{m}}$$

Adiabatic + field-like torque

$$\tilde{\mathbf{T}}_\mathrm{adia} = b_j \tilde{\partial}_x \hat{\mathbf{m}}$$

Nonadiabatic + damping-like torque

$$\tilde{\mathbf{T}}_\mathrm{non} = -\beta b_j \hat{\mathbf{m}} \times \tilde{\partial}_x \hat{\mathbf{m}}$$

$$\tilde{\partial}_x \hat{\mathbf{m}} \equiv \left(\partial_x + \frac{2\alpha_\mathrm{R} m_e}{\hbar^2} \hat{\mathbf{y}} \times \right) \hat{\mathbf{m}} = \partial_x \hat{\mathbf{m}} + \frac{2\alpha_\mathrm{R} m_e}{\hbar^2} \hat{\mathbf{y}} \times \hat{\mathbf{m}}$$

Effects of Interfacial SOC (through 2D Rashba model Hamiltonian)

"Magnetic interfaces (inversion asymmetry) + Spin-Orbit coupling"

2D Rashba Hamiltonian

$$\mathcal{H} = \mathcal{H}_{\mathrm{kin}} + \mathcal{H}_R + \mathcal{H}_{\mathrm{exc}} + \mathcal{H}_{\mathrm{imp}} = \frac{\mathbf{p}^2}{2m_e} + \frac{\alpha_{\mathrm{R}}}{\hbar}\boldsymbol{\sigma}\cdot(\mathbf{p}\times\hat{\mathbf{z}}) + J\boldsymbol{\sigma}\cdot\hat{\mathbf{m}} + \mathcal{H}_{\mathrm{imp}},$$

Unitary transformation

$$\mathcal{U}^{\dagger}\mathcal{H}\,\mathcal{U} = \mathcal{H}_{\mathrm{kin}} + J\boldsymbol{\sigma}\cdot\hat{\mathbf{m}}' + \mathcal{H}'_{\mathrm{imp}} + \mathcal{O}(\alpha_R^2),$$

where

$$\hat{\mathbf{m}}' = \mathcal{R}^{-1}\hat{\mathbf{m}}$$

K.-W. Kim et al. PRL 111, 216601 (2013)

Chiral derivative

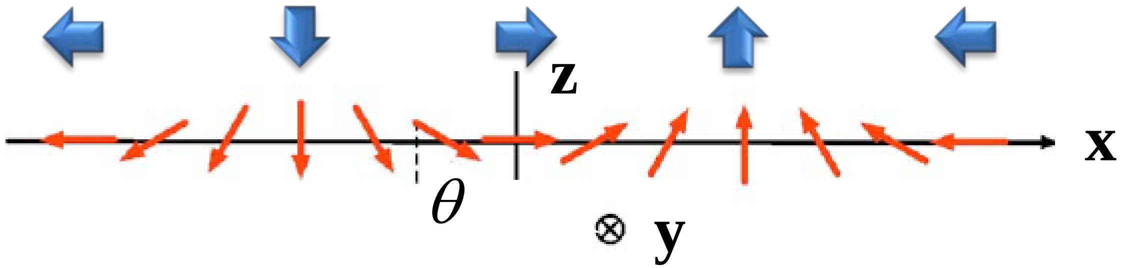
$$\tilde{\partial}_x\hat{\mathbf{m}} \equiv \left(\partial_x + \frac{2\alpha_{\mathrm{R}}m_e}{\hbar^2}\hat{\mathbf{y}}\times\right)\hat{\mathbf{m}} = \partial_x\hat{\mathbf{m}} + \frac{2\alpha_{\mathrm{R}}m_e}{\hbar^2}\hat{\mathbf{y}}\times\hat{\mathbf{m}}$$

"Chiral" derivative

K.-W. Kim et al. PRL 111, 216601 (2013)

$$\tilde{\partial}_x \hat{\mathbf{m}} \equiv \left(\partial_x + \frac{2\alpha_R m_e}{\hbar^2} \hat{\mathbf{y}} \times \right) \hat{\mathbf{m}} = \partial_x \hat{\mathbf{m}} + \frac{2\alpha_R m_e}{\hbar^2} \hat{\mathbf{y}} \times \hat{\mathbf{m}}$$

$$\tilde{\partial}_x \hat{\mathbf{m}} = 0$$



$$\frac{d\theta}{dx} = \frac{2\alpha_R m_e}{\hbar^2}$$

= precession rate of conduction electron spins due to RSOC

- ➔ No current-induced torque if magnetization precesses at the same rate as the conduction electrons would due to RSOC
- ➔ RSOC induces the chirality in the spin torques
- ➔ "Chiral" derivative captures RSOC effects

Exchange "stiffness" energy

$$A\int dV \partial_x \hat{\mathbf{m}} \cdot \partial_x \hat{\mathbf{m}}$$

Exchange "stiffness" interaction mediated by conduction electrons

- Conduction electron mediation

$$A\int dV \partial_x \hat{\mathbf{m}} \cdot \partial_x \hat{\mathbf{m}}$$

⬇ RSOC

$$A\int dV \tilde{\partial}_x \hat{\mathbf{m}} \cdot \tilde{\partial}_x \hat{\mathbf{m}}$$

Continuum version of interfacial Dzyaloshinskii-Moriya interaction (DMI)

$$= A\int dV \partial_x \hat{\mathbf{m}} \cdot \partial_x \hat{\mathbf{m}} + A\frac{4\alpha_{\mathrm{R}} m_e}{\hbar^2}\int dV \hat{\mathbf{y}} \cdot \hat{\mathbf{m}} \times \partial_x \hat{\mathbf{m}}$$

$$+ O\left(\alpha_{\mathrm{R}}\right)^2$$

A simple picture: why RSOC ⇔ DMI

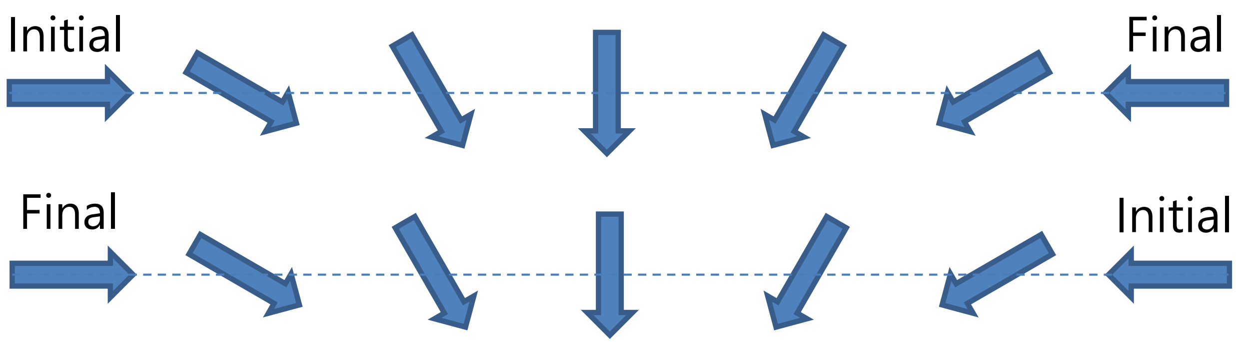
- DMI = Current-INDEPENDENT magnetic energy

➔ Contributions from spins with +k + Cont. from spins with –k **≠ 0**

- RSOC ➔ effective B field in **k** x (interface normal)

k > 0 ➔ B ⊙

k < 0 ➔ B ⊗



➔ Conduction electron spins are chiral

➔ + s-d exchange

➔ Localized electron spins (**m**) tend to be chiral

Spin motive force (SMF)

$$\mathbf{E}_{\pm} = \pm \sum_{i=x,y} \hat{\mathbf{x}}_i \frac{\hbar}{2e} \partial_t \hat{\mathbf{m}} \times \partial_i \hat{\mathbf{m}} \cdot \hat{\mathbf{m}}$$

Berger, PRB 1986
Volovik, JPC 1987
Barnes & Maekawa, PRL 2007

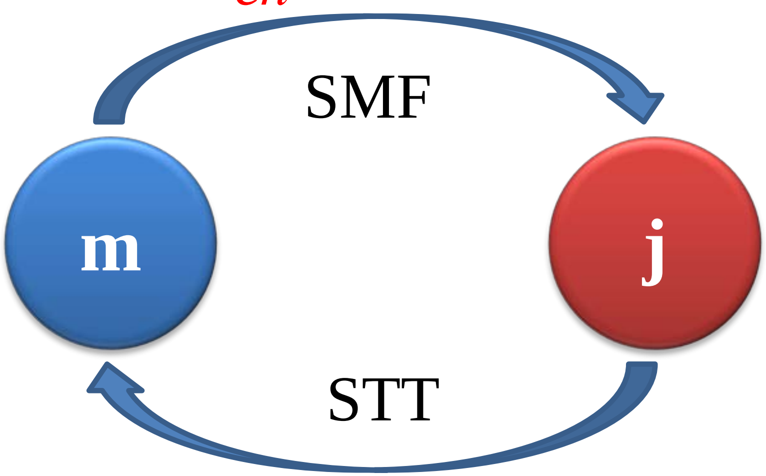
With RSOC

$$\tilde{\mathbf{E}}_{\pm} = \pm \sum_{i=x,y} \hat{\mathbf{x}}_i \frac{\hbar}{2e} \partial_t \hat{\mathbf{m}} \times \tilde{\partial}_i \hat{\mathbf{m}} \cdot \hat{\mathbf{m}}$$

$$= \pm \sum_{i=x,y} \hat{\mathbf{x}}_i \frac{\hbar}{2e} \partial_t \hat{\mathbf{m}} \times \partial_i \hat{\mathbf{m}} \cdot \hat{\mathbf{m}} \pm \sum_{i=x,y} \hat{\mathbf{x}}_i \frac{\hbar}{2e} \partial_t \hat{\mathbf{m}} \times \left[\frac{2\alpha_R m_e}{\hbar^2} \left(\hat{\mathbf{z}} \times \hat{\mathbf{x}}_i \right) \times \hat{\mathbf{m}} \right] \cdot \hat{\mathbf{m}}$$

$$= \mathbf{E}_{\pm} \pm \frac{\alpha_R m_e}{e\hbar} \hat{\mathbf{z}} \times \partial_t \hat{\mathbf{m}}$$

Kim, Moon, KJL, HWL, PRL 2012



Foros et al. PRB '08
Tserkovnyak & Wong, PRB '09
Zhang & Zhang, PRL 2009

➔ Feedback-induced modification of Landau-Lifshitz-Gilbert eq.

Feedback effect on LLG equation (1)

- Feedback-induced damping torque

$$\mathbf{T}_{\text{feedback}} = \mathbf{m} \times \delta\tilde{\alpha}_{\text{G}} \cdot \frac{\partial \mathbf{m}}{\partial t}$$

$$(\delta\tilde{\alpha}_{\text{G}})_{ij} = \eta \sum_k \left(X_{ki} + \frac{2\alpha_{\text{R}} m_e}{\hbar} \varepsilon_{3ki} \right)\left(X_{kj} + \frac{2\alpha_{\text{R}} m_e}{\hbar} \varepsilon_{3kj} \right), \quad X_{ki} = \left(\mathbf{m} \times \frac{\partial \mathbf{m}}{\partial x_k} \right)_i, \quad \eta = \frac{\mu_{\text{B}} \hbar \sigma_{\text{c}}}{2e^2 M_{\text{s}}}$$

Zhang & Zhang, PRL '09 — Kim, Moon, KJL, HWL, PRL 2012

Single domain

$$\tilde{\alpha} = \begin{pmatrix} \alpha_0 + C\alpha_R^2 & 0 & 0 \\ 0 & \alpha_0 + C\alpha_R^2 & 0 \\ 0 & 0 & \alpha_0 \end{pmatrix}$$

Domain wall

$$\hat{\mathbf{m}} = \left(\cos\phi \,\mathbf{sech}\left(\frac{x}{\lambda}\right), \sin\phi \,\mathbf{sech}\left(\frac{x}{\lambda}\right), \ \mathbf{tanh}\left(\frac{x}{\lambda}\right) \right)$$

$$\tilde{\alpha}_{22} \approx \alpha_0 - \frac{C}{\lambda} \cos\phi \,\mathbf{sech}\left(\frac{x}{\lambda}\right) \alpha_R + O(\alpha_R^2)$$

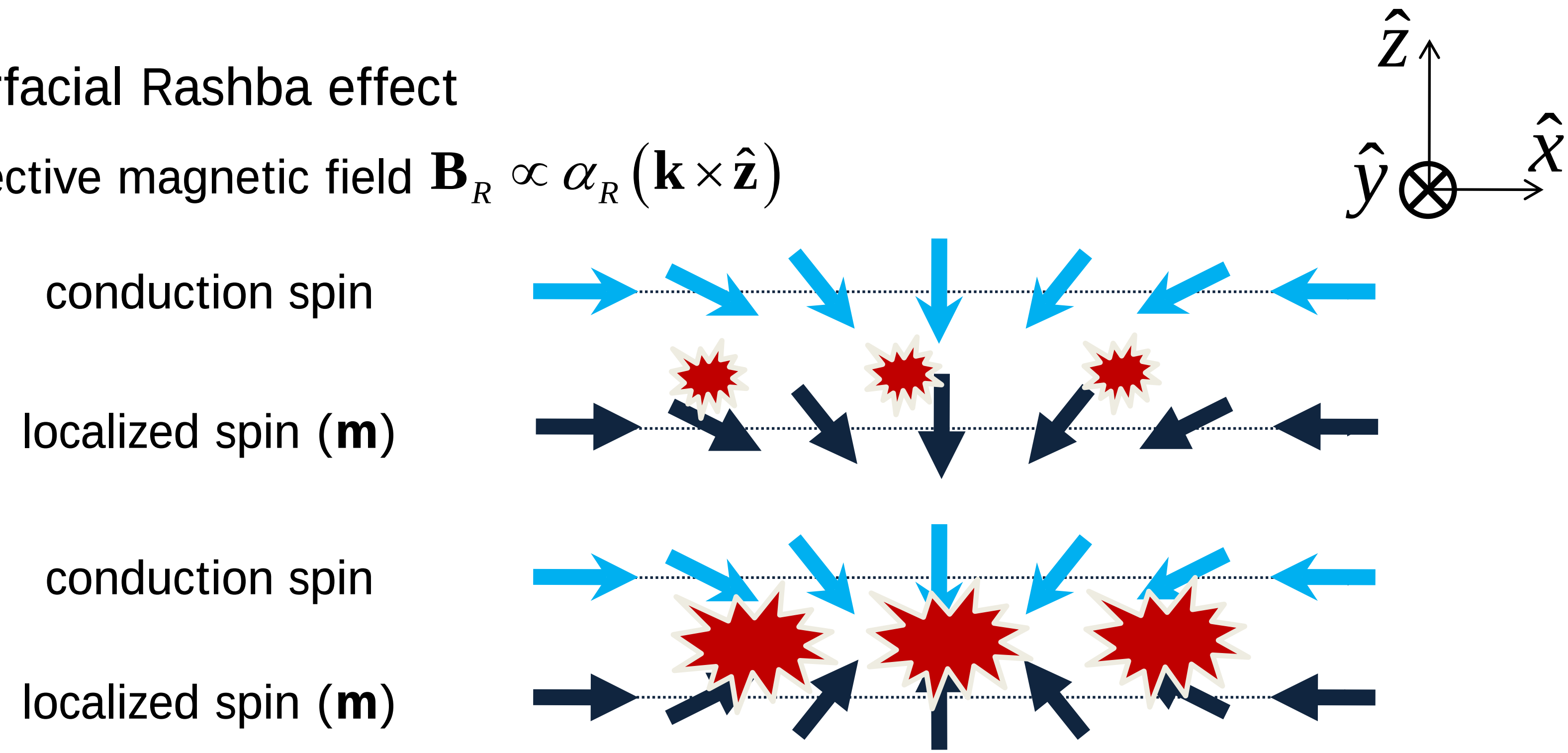
➔ Anisotropic enhanced Gilbert damping

➔ **Magnetic texture-dependent Chiral damping**

Chiral damping

- Interfacial Rashba effect

  Effective magnetic field $\mathbf{B}_R \propto \alpha_R (\mathbf{k} \times \hat{\mathbf{z}})$



: Non-compensated chirality of conduction electron spin

➔ Chirality of localized electron spin due to s-d exchange coupling

(Dzyaroshinskii-Moriya interaction)

: More frequent scattering ➔ chiral damping

Spin-dependent magnetic field

$$\mathbf{B}_{\pm} = \mp \hat{\mathbf{z}} \frac{\hbar}{2e} \left(\partial_x \hat{\mathbf{m}} \times \partial_y \hat{\mathbf{m}} \right) \cdot \hat{\mathbf{m}}$$

Volovik, JPC 1987

With RSOC

RSOC contribution to anomalous Hall effect (Bijl-Duine PRB '12)

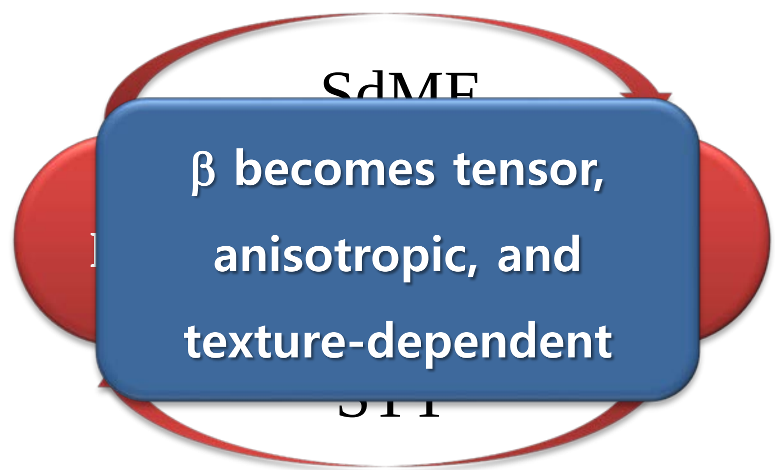
$$\tilde{\mathbf{B}}_{\pm} = \mp \hat{\mathbf{z}} \frac{\hbar}{2e} \left(\tilde{\partial}_x \hat{\mathbf{m}} \times \tilde{\partial}_y \hat{\mathbf{m}} \right) \cdot \hat{\mathbf{m}}$$

$$= \mp \hat{\mathbf{z}} \frac{\hbar}{2e} \left(\partial_x \hat{\mathbf{m}} \times \partial_y \hat{\mathbf{m}} \right) \cdot \hat{\mathbf{m}} \pm \frac{\alpha_{\mathrm{R}} m_e}{e\hbar} \nabla \times \left(\hat{\mathbf{z}} \times \hat{\mathbf{m}} \right) \pm \frac{2\alpha_{\mathrm{R}}^2 m_e^2}{e\hbar^3} m_z \hat{\mathbf{z}}$$

Topological Hall Effect

(2nd Rashba term)/(1st Conv. Term) ~ 1

Spin spirals can be detected

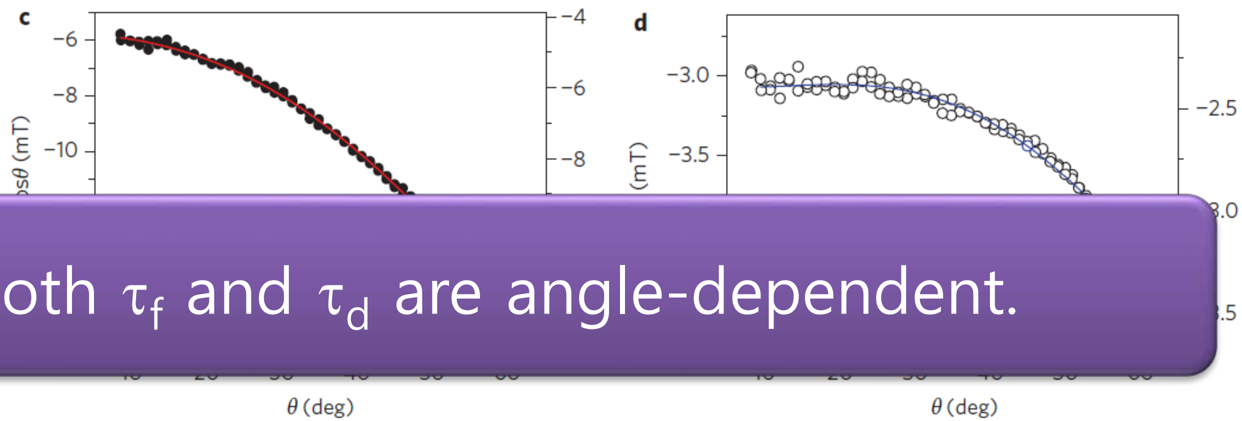


β becomes tensor, anisotropic, and texture-dependent

Angular dependence of SOTs: Previous Experiments

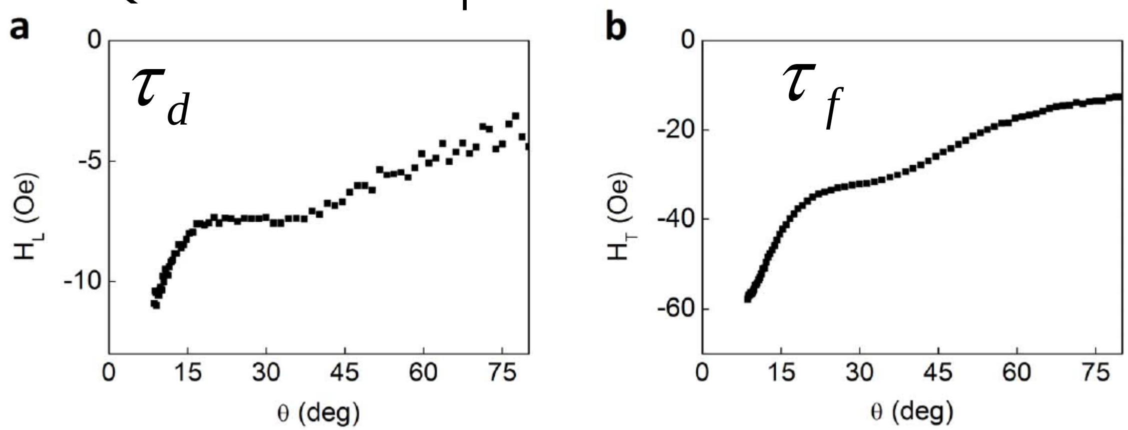
$$\mathbf{SOT} = \tau_f \hat{\mathbf{m}} \times \hat{\mathbf{y}} + \tau_d \hat{\mathbf{m}} \times (\hat{\mathbf{m}} \times \hat{\mathbf{y}})$$

Experiment: Garello et al. Nat. Nanotech. 2013



- Both τ_f and τ_d are angle-dependent.

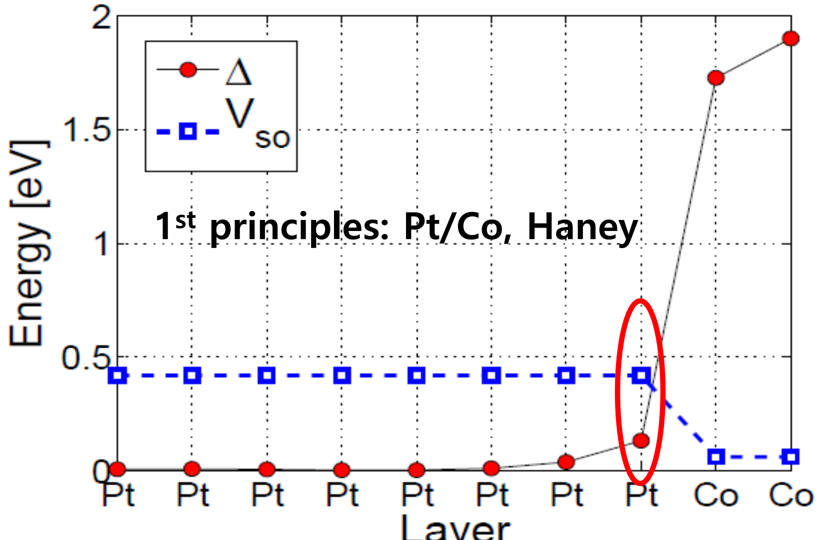
Experiment: Qiu et al. Sci. Rep. 2013



Angular dependence of SOTs: Previous Theories

$$\mathbf{SOT} = \tau_f \hat{\mathbf{m}} \times \hat{\mathbf{y}} + \tau_d \hat{\mathbf{m}} \times (\hat{\mathbf{m}} \times \hat{\mathbf{y}})$$

- Bulk SOC in HM (SHE: Haney et al. PRB '13)
  - ➔ No angular dependence (1^{st} order of SH angle)
- Interfacial SOC (Rashba: Pauyac et al. APL '13)
  - ➔ Weak Rashba regimes: both are almost constant
  - ➔ Strong Rashba regimes: τ_f = constant, τ_d = angle-dependent



Berry phase contribution to τ_d (Kurebayashi et al. Nat. Nano. '14)

Fermi surface distortion due to Rashba SOC

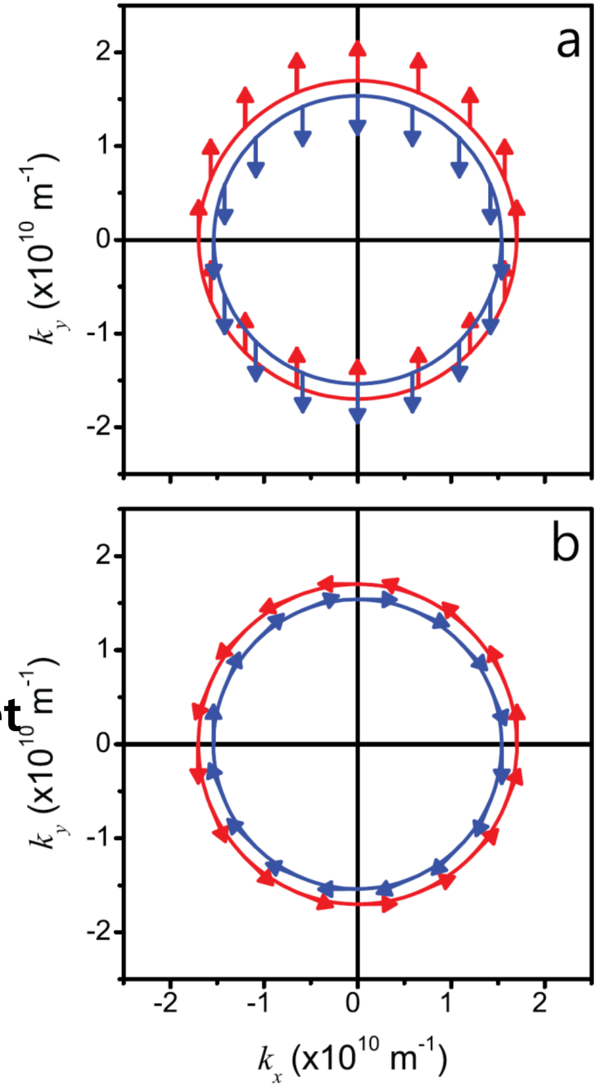
Ferromagnet

$\alpha_R = 0$ & $J_{sd} \neq 0$

M = (0, 1, 0)

Rashba non-magnet

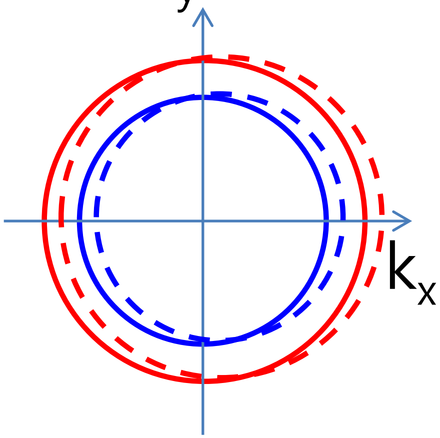
$\alpha_R \neq 0$ & $J_{sd} = 0$



SOT: Fermi surface contribution ➔ τ_f

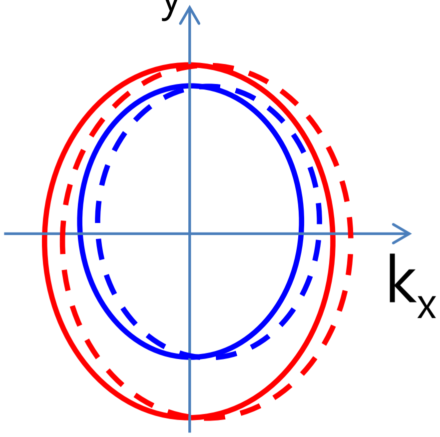
E-field

$\mathbf{M} = M_z$



Circle & Concentric

$\mathbf{M} = M_x$



Ellipse & Non-concentric

$\tau_f(M_z) \neq \tau_f(M_x)$

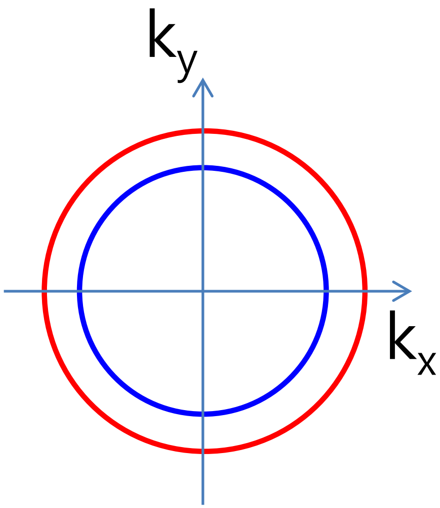
SOT: Fermi sea contribution (Berry phase) ➔ τ_d

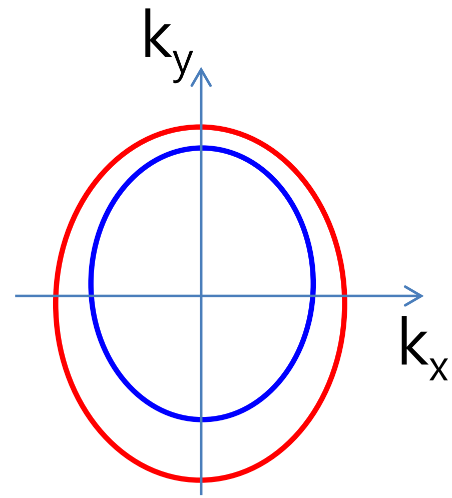
E-field

$$\mathbf{s}^{\pm}_{\mathbf{k}(2)} = \mp \frac{\hbar}{2}\alpha_{\mathrm{R}} e E \left(\frac{J_{\mathrm{sd}} m_z}{2\epsilon^3_{\mathbf{k}}}, 0, -\frac{\alpha_{\mathrm{R}} k_y + J_{sd} m_x}{2\epsilon^3_{\mathbf{k}}} \right)$$

M = M_z

M = M_x





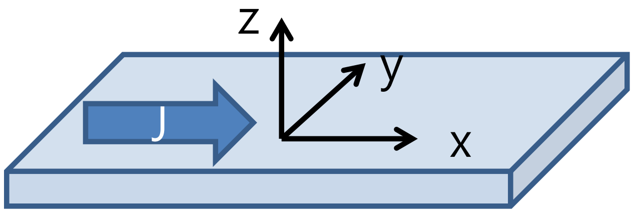
Circle & Concentric

Ellipse & Non-concentric

$\tau_d(M_z) \neq \tau_d(M_x)$

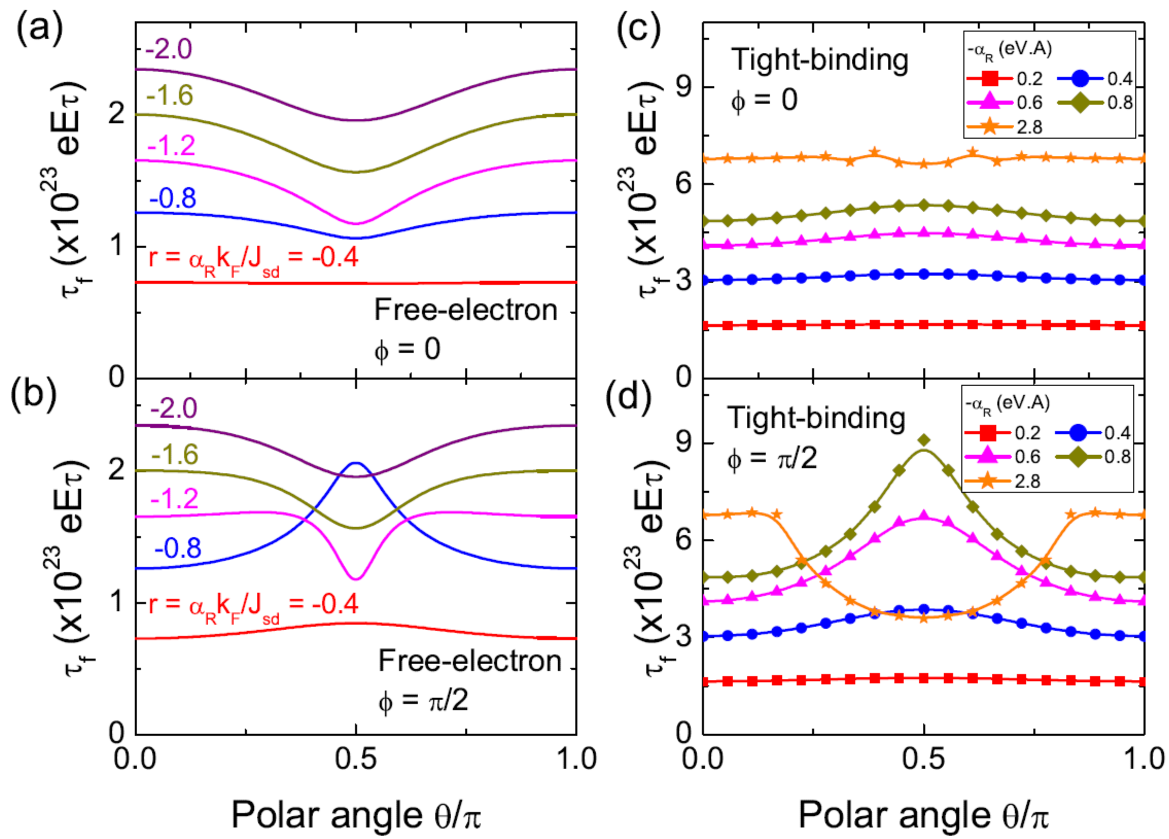
Field-like torque (τ_f): Fermi surface contribution

Lee et al. PRB 91, 144401 (2015)



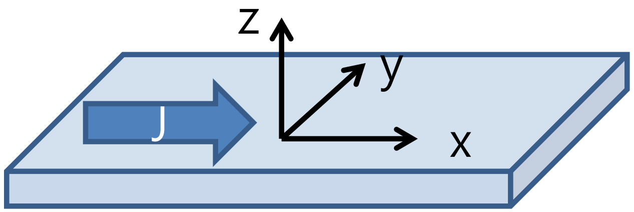
$$\hat{\mathbf{m}} = (\cos\phi \sin\theta, \sin\phi \sin\theta, \cos\theta)$$

E_F = 10 eV, J_{sd} = 1 eV, $m^* = m_0$



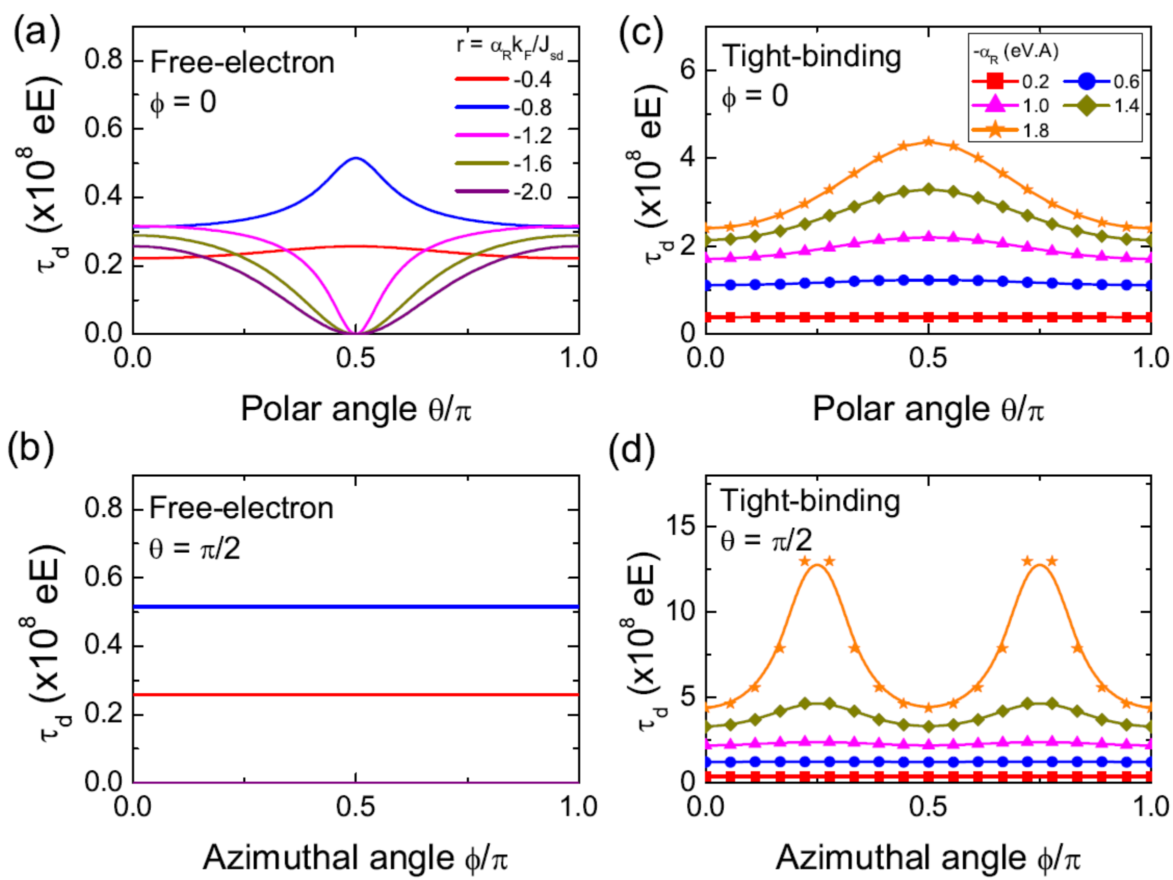
Strong angular dependence near $\alpha_R k_F / J_{sd} = 1$ ➔ FS distortion

Damping-like torque (τ_d): Fermi sea contribution



$$\hat{\mathbf{m}} = (\cos\phi\sin\theta, \sin\phi\sin\theta, \cos\theta)$$

E_F = 10 eV, J_{sd} = 1 eV, $m^* = m_0$



Strong angular dependence near $\alpha_R k_F/J_{sd}$ = or > 1

Angle-dependent SOT: Switching and DW motion

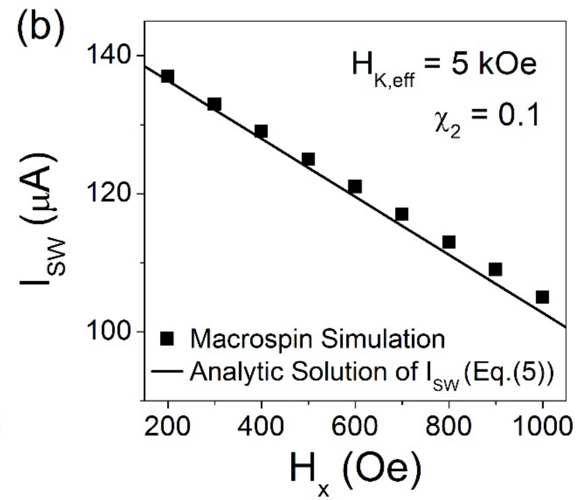
Seo-Won Lee and KJL, JKPS '15

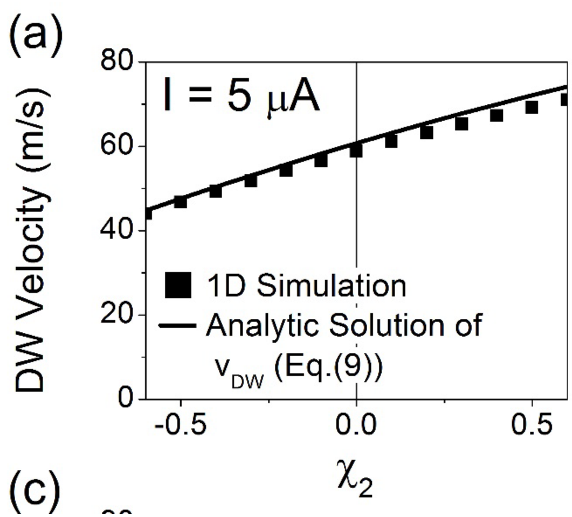
$$\frac{\partial \hat{\mathbf{m}}}{\partial t} = -\gamma \hat{\mathbf{m}} \times \mathbf{H}_{eff} + \alpha\, \hat{\mathbf{m}} \times \frac{\partial \hat{\mathbf{m}}}{\partial t} + \gamma \tau_{d0} \hat{\mathbf{m}} \times \left[(\hat{\mathbf{m}} \times \hat{\mathbf{y}}) + \boxed{\hat{\mathbf{z}} (\hat{\mathbf{m}} \cdot \hat{\mathbf{x}}) \left(\chi_2 + \chi_4 (\hat{\mathbf{m}} \times \hat{\mathbf{z}})^2 \right)} \right]$$

Angle-dependent part

Single domain switching

$$\tau_{d0}^{sw} = H_{K,eff} \left(\frac{\sqrt{\Gamma - h^4 + 20h^2}}{4\sqrt{2}} + \boxed{\frac{h^6 + (h^5 - 4h^3 + 12h)\sqrt{8 + h^2} + 12h^2 - 16}{16\sqrt{2}\sqrt{\Gamma - h^4 + 20h^2}} \chi_2} \right),$$





DW speed

$$v_{DW} = \gamma \frac{\pi}{2} \frac{\lambda D_0 \tau_{d0} \left(1 + \boxed{\frac{\chi_2}{2} + \frac{3}{8}\chi_4} \right)}{\sqrt{D_0^2 \alpha^2 + M_S^2 \tau_{d0}^2 \left(1 + \boxed{\frac{\chi_2}{2} + \frac{3}{8}\chi_4} \right)^2 \lambda^2}}.$$

Summary

1. Crucial role of SOC in magnetic properties and spin transport
2. Understanding of these SOC-related phenomena based on orbital moment
3. Rashbaness is always preserved for inversion symmetry broken systems
4. Orbital engineering is key to make SOC phenomena useful